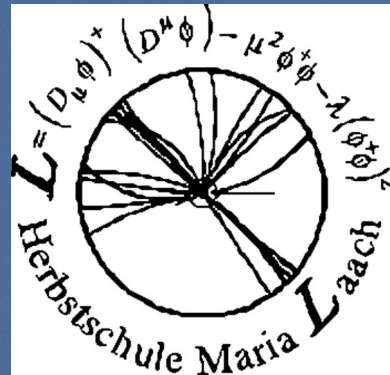
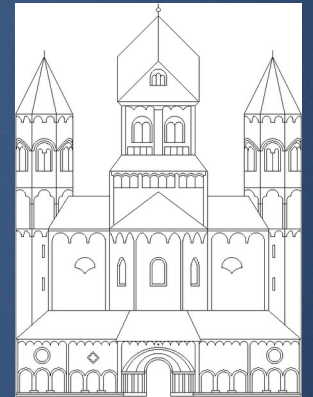


QCD and Jets at the LHC

V02 – From jet definition to prediction and measurement



Herbstschule
Maria Laach



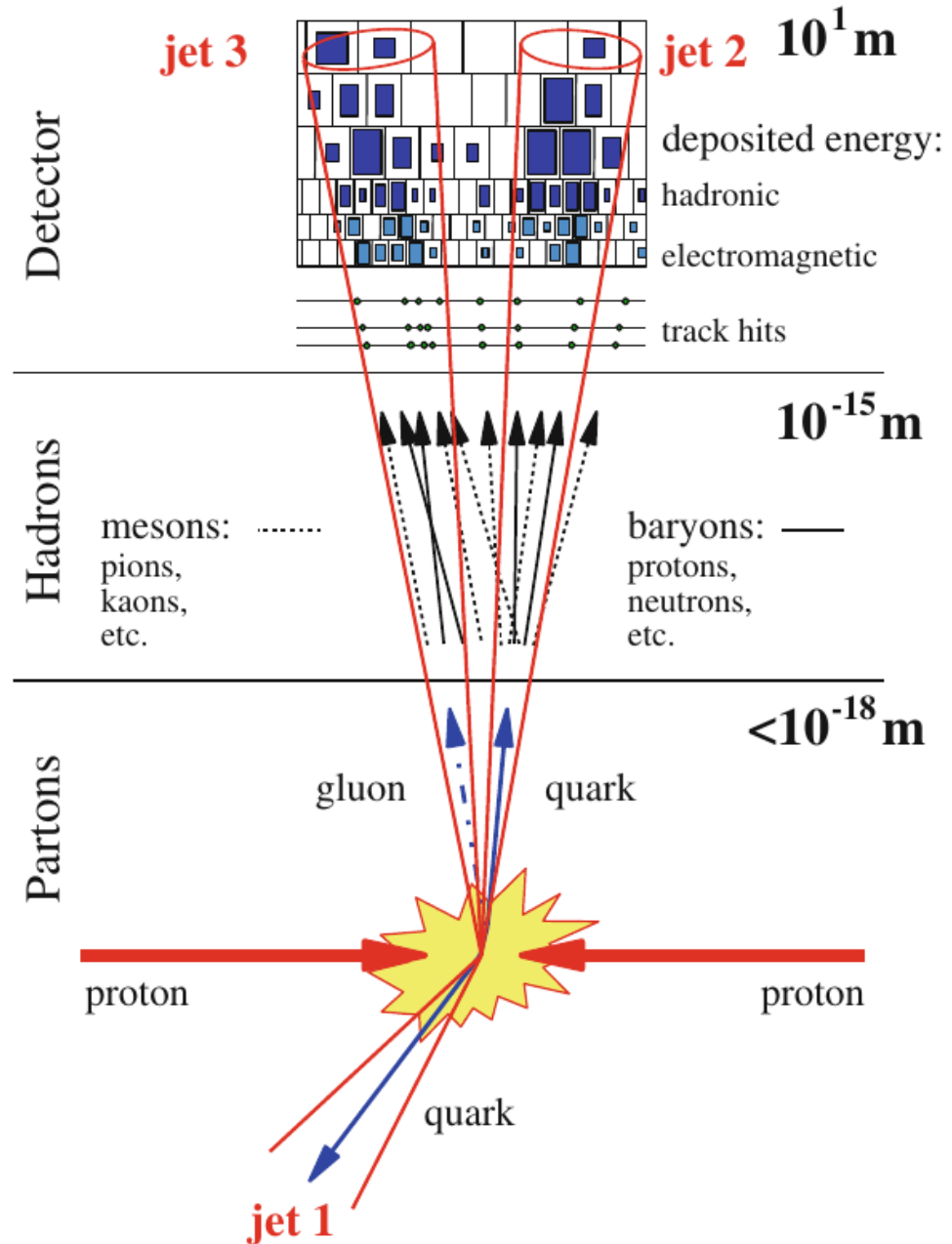
Outline



- **Jet algorithms at LHC**
- **DIS and PDFs**
- **Hadron-hadron collisions**
- **Jet reconstruction**



Tools in particle physics: Jets





Jet algorithms

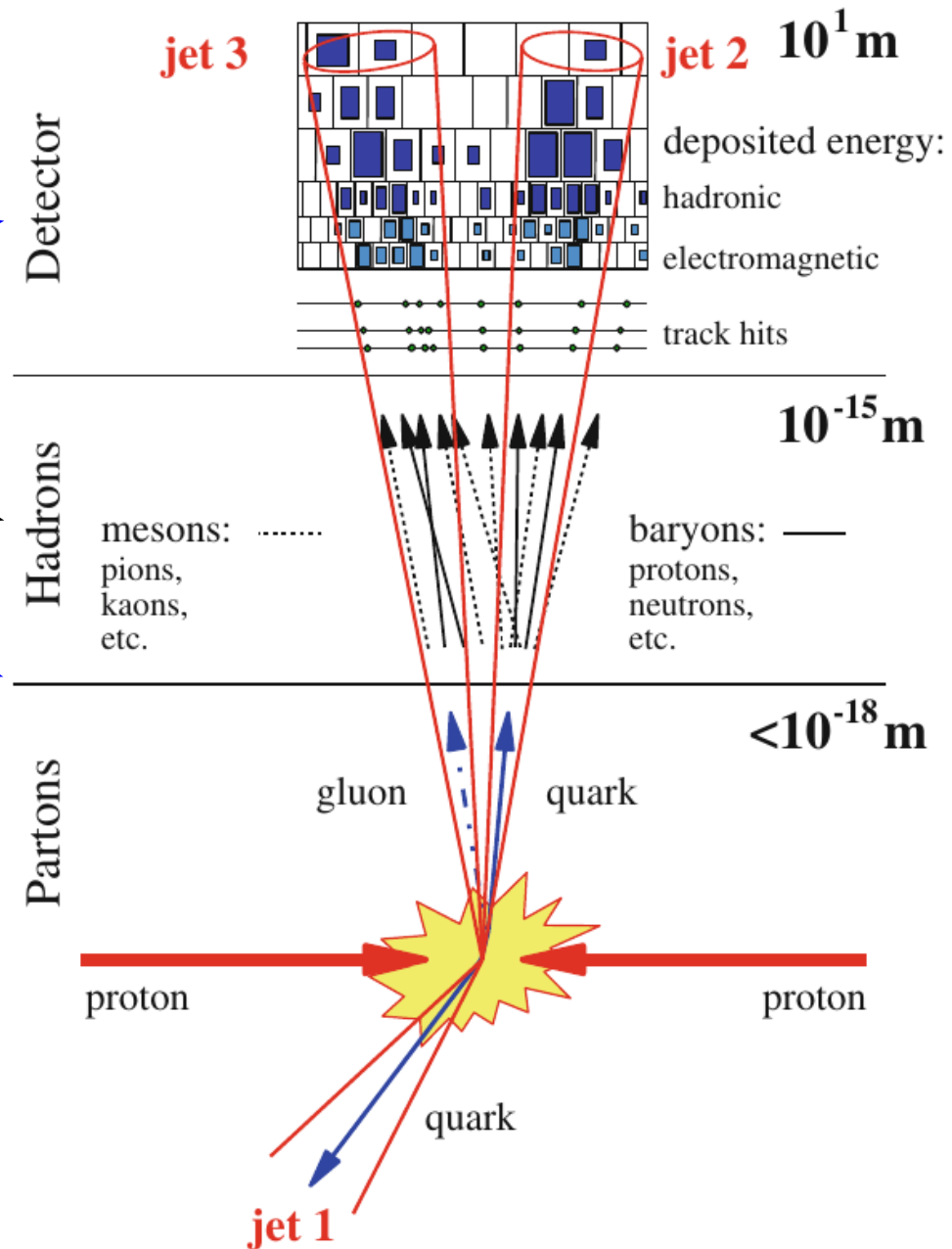
Primary goal:

Good correspondence among:

- **Detector measurements**
- **Particles in final state** and
- **"hard" partons**

Two classes of algorithms:

- 1. Cone algorithms:** "Geometrical" attribution of objects to the direction of largest energy flow in an event (First choice at **hadron colliders**)
- 2. Sequential recombination:** Iterated combination of closest neighbors among all pairs of objects (First choice at **e^+e^- & ep colliders**)





From e^+e^- to pp algorithm

• **No E_{vis} for normalisation**

+ **Drop E_{vis} , $y_{ij} \rightarrow d_{ij}$ has dimensions**

$$y_{ij}^{\text{kT}} = \frac{2 \min(E_i^2, E_j^2)(1 - \cos(\theta_{ij}))}{E_{\text{vis}}^2} \quad e^+e^-$$

$$d_{ij} = 2 \min(E_i^2, E_j^2)(1 - \cos(\theta_{ij}))$$



From e^+e^- to pp algorithm



- **No E_{vis} for normalisation**
 - + **Drop E_{vis} , $y_{ij} \rightarrow d_{ij}$ has dimensions**
- **Account for beam remnants**
 - + **2nd distance measure**

$$y_{ij}^{\text{kT}} = \frac{2 \min(E_i^2, E_j^2)(1 - \cos(\theta_{ij}))}{E_{\text{vis}}^2}$$

e^+e^-

$$d_{ij} = 2 \min(E_i^2, E_j^2)(1 - \cos(\theta_{ij}))$$

$$d_{iB} = 2E_i^2(1 - \cos(\theta_{iB}))$$



From e^+e^- to pp algorithm

- **No E_{vis} for normalisation**
 - + **Drop E_{vis} , $y_{ij} \rightarrow d_{ij}$ has dimensions**
- **Account for beam remnants**
 - + **2nd distance measure**
- **Longitudinal boost invariance**
 - + **Only use p_{T} , rapidity y , azimuth Φ**

$$y_{ij}^{\text{kT}} = \frac{2 \min(E_i^2, E_j^2)(1 - \cos(\theta_{ij}))}{E_{\text{vis}}^2} \quad e^+e^-$$

$$d_{ij} = 2 \min(E_i^2, E_j^2)(1 - \cos(\theta_{ij}))$$

$$d_{iB} = 2E_i^2(1 - \cos(\theta_{iB}))$$

$$d_{ij} = \min(p_{ti}^2, p_{tj}^2) \cdot \Delta R_{ij}^2 \quad d_{iB} = p_{ti}^2$$

$$\Delta R_{ij}^2 = \Delta\phi_{ij}^2 + \Delta y_{ij}^2$$



From e^+e^- to pp algorithm

- **No E_{vis} for normalisation**
 - + **Drop E_{vis} , $y_{ij} \rightarrow d_{ij}$ has dimensions**
- **Account for beam remnants**
 - + **2nd distance measure**
- **Longitudinal boost invariance**
 - + **Only use p_{T} , rapidity y , azimuth Φ**
- **No absolute scale to define d_{cut}**
 - + **Replace y_{cut} by angular size R**

$$y_{ij}^{\text{kT}} = \frac{2 \min(E_i^2, E_j^2)(1 - \cos(\theta_{ij}))}{E_{\text{vis}}^2} \quad e^+e^-$$

$$d_{ij} = 2 \min(E_i^2, E_j^2)(1 - \cos(\theta_{ij}))$$

$$d_{iB} = 2E_i^2(1 - \cos(\theta_{iB}))$$

$$d_{ij} = \min(p_{ti}^2, p_{tj}^2) \cdot \Delta R_{ij}^2 \quad d_{iB} = p_{ti}^2$$

$$d_{ij} = \min(p_{ti}^2, p_{tj}^2) \cdot \frac{\Delta R_{ij}^2}{R^2} \quad d_{iB} = p_{ti}^2$$

$$\Delta R_{ij}^2 = \Delta\phi_{ij}^2 + \Delta y_{ij}^2$$



From e^+e^- to pp algorithm



• No E_{vis} for normalisation

$$y_{ij}^{\text{kT}} = \frac{2 \min(E_i^2, E_j^2)(1 - \cos(\theta_{ij}))}{E_{\text{vis}}^2} \quad e^+e^-$$

➔ Drop E_{vis} , $y_{ij} \rightarrow d_{ij}$ has dimensions

• Account for beam remnants

$$d_{ij} = 2 \min(E_i^2, E_j^2)(1 - \cos(\theta_{ij}))$$

➔ 2nd distance measure

$$d_{iB} = 2E_i^2(1 - \cos(\theta_{iB}))$$

• Longitudinal boost invariance

➔ Only use p_{T} , rapidity y , azimuth Φ

$$d_{ij} = \min(p_{ti}^2, p_{tj}^2) \cdot \Delta R_{ij}^2 \quad d_{iB} = p_{ti}^2$$

• No absolute scale to define d_{cut}

➔ Replace y_{cut} by angular size R

$$d_{ij} = \min(p_{ti}^2, p_{tj}^2) \cdot \frac{\Delta R_{ij}^2}{R^2} \quad d_{iB} = p_{ti}^2$$

• Sequential recombination:

- ➔ 1. find smallest of all d_{ij} , d_{iB} in object list
- ➔ 2. if d_{ij} , replace i & j by combined list entry
- ➔ 3. if d_{iB} , declare i a jet & remove from list
- ➔ 4. iterate until only jets left

$$\Delta R_{ij}^2 = \Delta\phi_{ij}^2 + \Delta y_{ij}^2$$



From e^+e^- to pp algorithm



• No E_{vis} for normalisation

$$y_{ij}^{k_T} = \frac{2 \min(E_i^2, E_j^2)(1 - \cos(\theta_{ij}))}{E_{\text{vis}}^2} \quad e^+e^-$$

➔ Drop E_{vis} , $y_{ij} \rightarrow d_{ij}$ has dimensions

• Account for beam remnants

$$d_{ij} = 2 \min(E_i^2, E_j^2)(1 - \cos(\theta_{ij}))$$

➔ 2nd distance measure

$$d_{iB} = 2E_i^2(1 - \cos(\theta_{iB}))$$

• Longitudinal boost invariance

➔ Only use p_T , rapidity y , azimuth Φ

$$d_{ij} = \min(p_{ti}^2, p_{tj}^2) \cdot \Delta R_{ij}^2 \quad d_{iB} = p_{ti}^2$$

• No absolute scale to define d_{cut}

➔ Replace y_{cut} by angular size R

$$d_{ij} = \boxed{\min(p_{ti}^2, p_{tj}^2)} \cdot \frac{\Delta R_{ij}^2}{R^2} \quad d_{iB} = p_{ti}^2$$

• Sequential recombination:

➔ 1. find smallest of all d_{ij} , d_{iB} in object list

Inclusive k_T algorithm pp

➔ 2. if d_{ij} , replace i & j by combined list entry

➔ 3. if d_{iB} , declare i a jet & remove from list

➔ 4. iterate until only jets left

$$\boxed{\Delta R_{ij}^2 = \Delta\phi_{ij}^2 + \Delta y_{ij}^2}$$



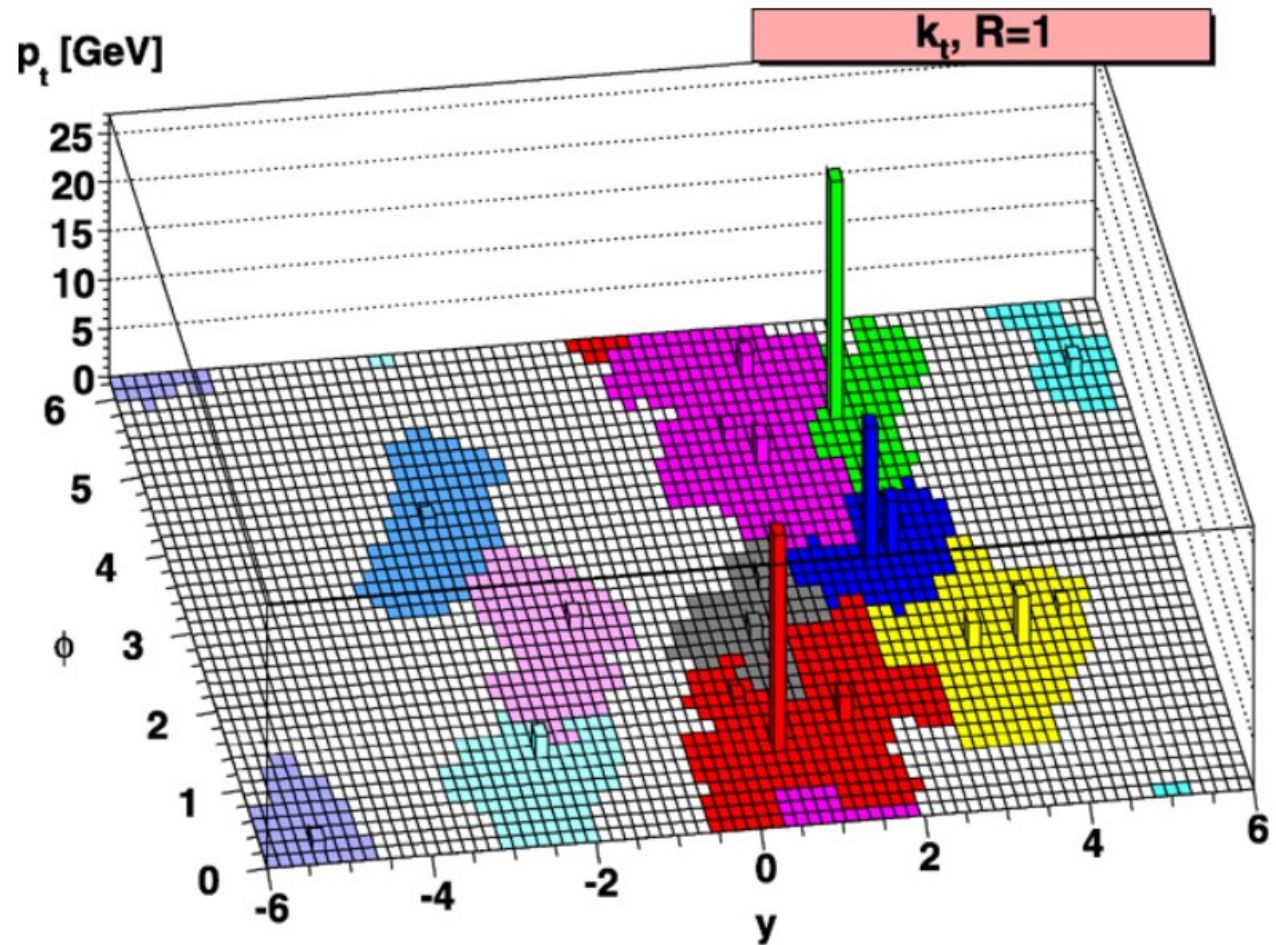
Inclusive k_T

Character sheet

- ➔ Clusters softest particles first
- ➔ Irregular shape in y, Φ plane
- ➔ Difficult to calibrate
- ➔ Undoes QCD splittings
- ➔ Meaningful clustering sequence
- ➔ Suited for substructure analysis
- ➔ Time per N particles $\sim N \ln N$
(originally thought to be $\sim N^3$)

Cacciari, Salam, PLB 641 (2006) 57.

$$d_{ij} = \min(p_{ti}^2, p_{tj}^2) \cdot \frac{\Delta R_{ij}^2}{R^2} \quad d_{iB} = p_{ti}^2$$



Ellis, Soper, PRD 48 (1993) 3160.



Inclusive k_T

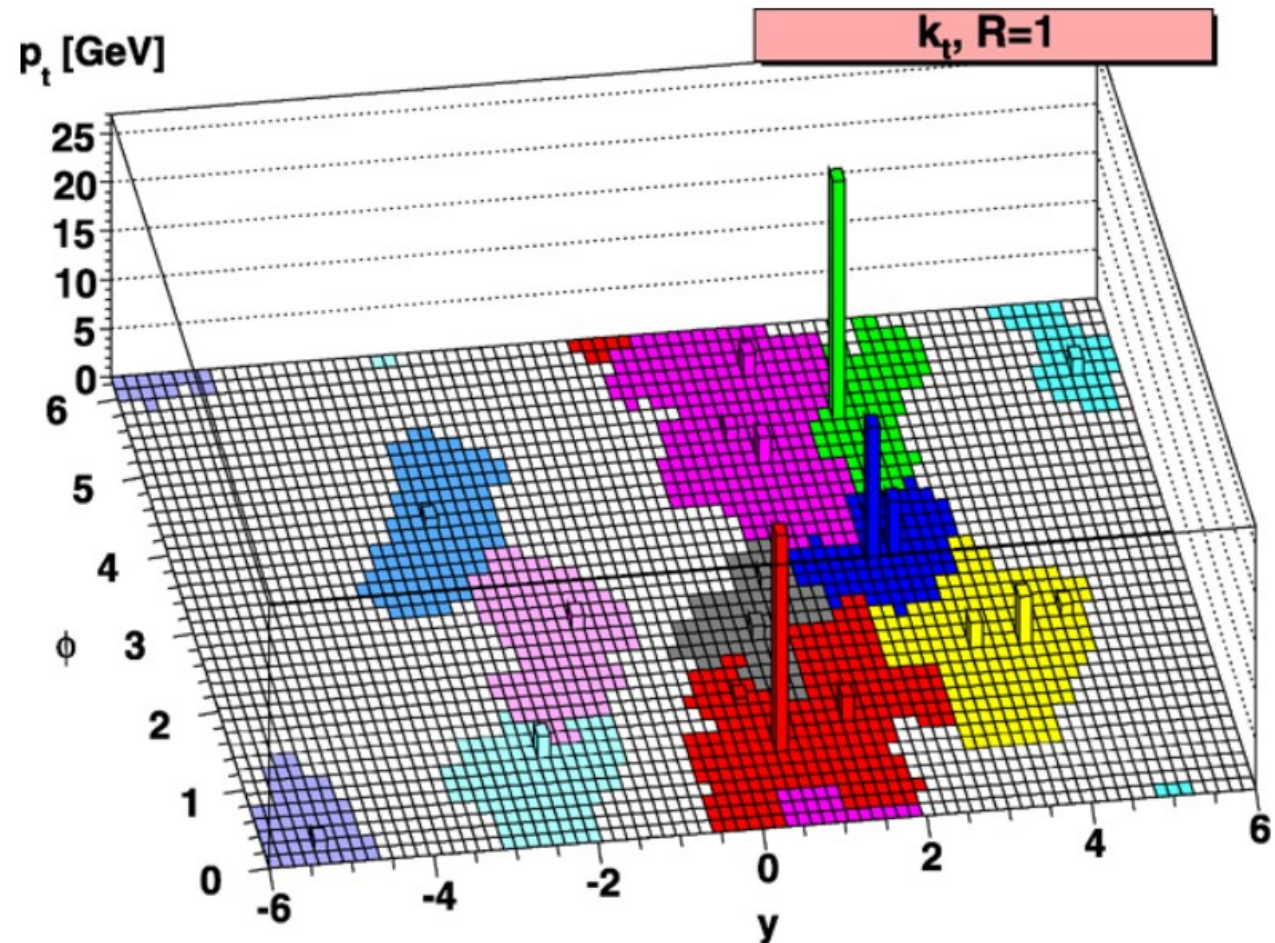
Character sheet

- ➔ Clusters softest particles first
- ➔ Irregular shape in y, Φ plane
- ➔ Difficult to calibrate
- ➔ Undoes QCD splittings
- ➔ Meaningful clustering sequence
- ➔ Suited for substructure analysis
- ➔ Time per N particles $\sim N \ln N$

Remark:

For such plots tens of thousands of $p_T \sim 0$ dust or ghost particles are distributed over the y, Φ plane and clustered into jets. This determines the coloring of the “jet areas” and only works with CI safe algorithms!

$$d_{ij} = \min(p_{ti}^2, p_{tj}^2) \cdot \frac{\Delta R_{ij}^2}{R^2} \quad d_{iB} = p_{ti}^2$$



Cacciari, Salam, Soyez, JHEP04 (2008) 005.

Ellis, Soper, PRD 48 (1993) 3160.



Character sheet

- ➔ Clusters particles closest in R
- ➔ Irregular shape in y, Φ plane
- ➔ Difficult to calibrate
- ➔ Meaningful clustering sequence
- ➔ Suited for substructure analysis
- ➔ Time per N particles $\sim N \ln N$

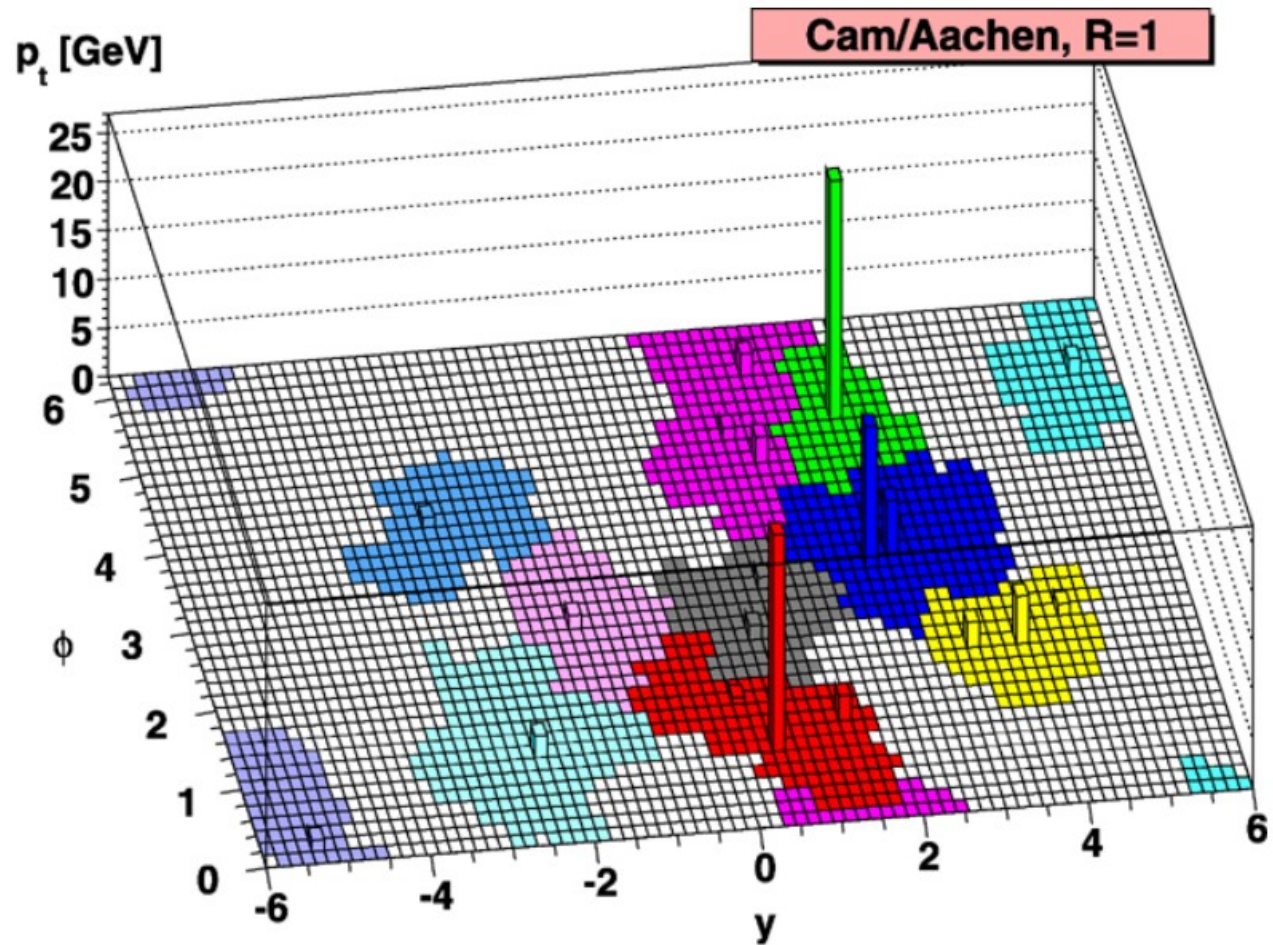
$$d_{ij} = \frac{\Delta R_{ij}^2}{R^2}$$

$$d_{iB} = 1$$

Alternative:

- ➔ p_T dropped in distance
- ➔ Strictly angular ordering

$$p_{ti}^2 \rightarrow p_{ti}^0 = 1$$



Dokshitzer et al., JHEP 08 (1997) 001,
Wobisch et al., Proceedings, MC Generators for HERA Physics (1998).



Character sheet

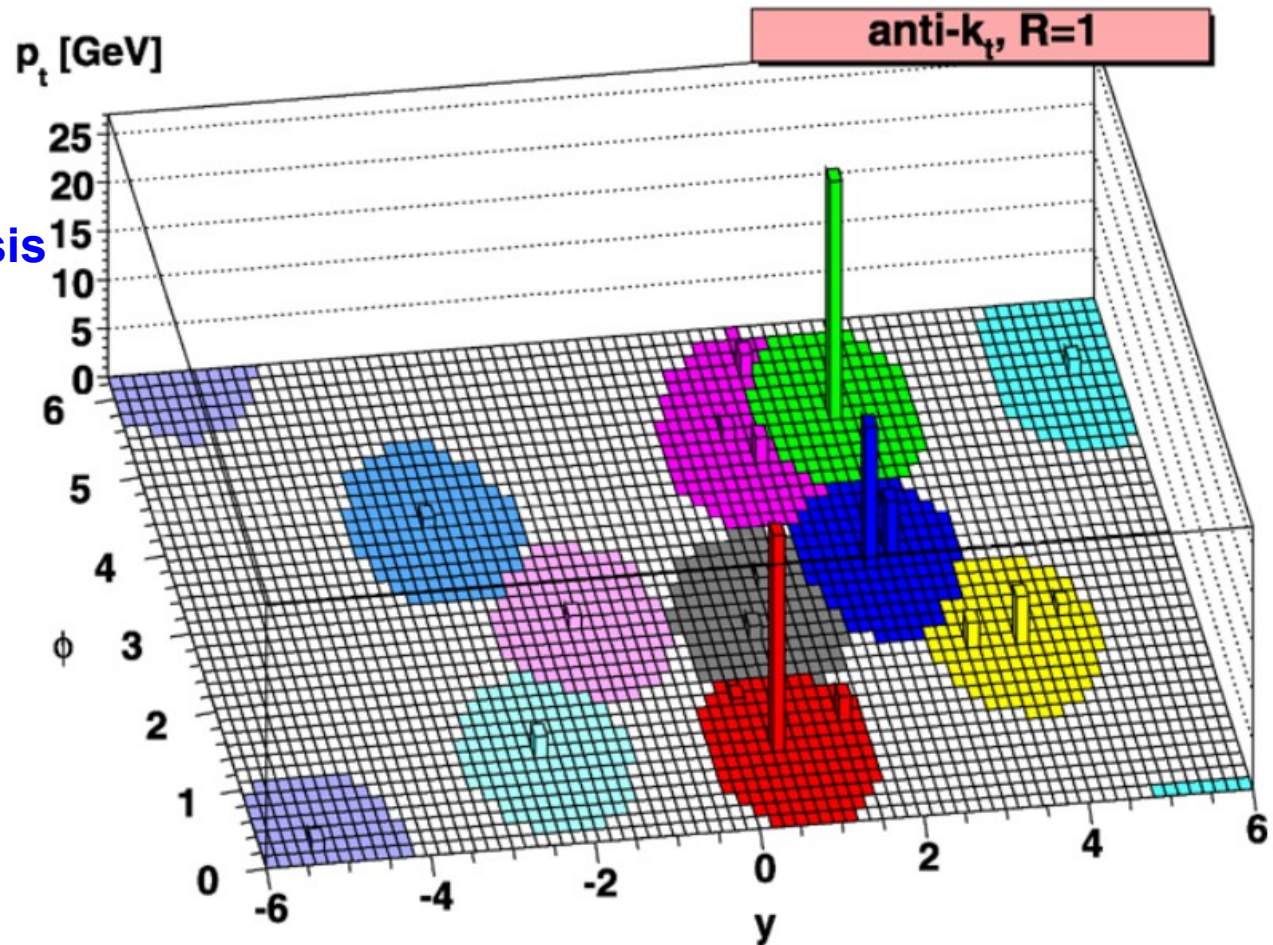
- ➔ Clusters hardest particles first
- ➔ Cone-like shape in y, Φ plane
- ➔ Simpler to calibrate
- ➔ Clustering sequence not useful
- ➔ Unsuitd for substructure analysis
- ➔ Time per N particles $\sim N^{3/2}$

$$d_{ij} = \min(p_{ti}^{-2}, p_{tj}^{-2}) \cdot \frac{\Delta R_{ij}^2}{R^2} \quad d_{iB} = p_{ti}^{-2}$$

Alternative:

- ➔ Use $1 / p_T^2$ in distance

$$p_{ti}^2 \rightarrow p_{ti}^{-2} = \frac{1}{p_{ti}^2}$$



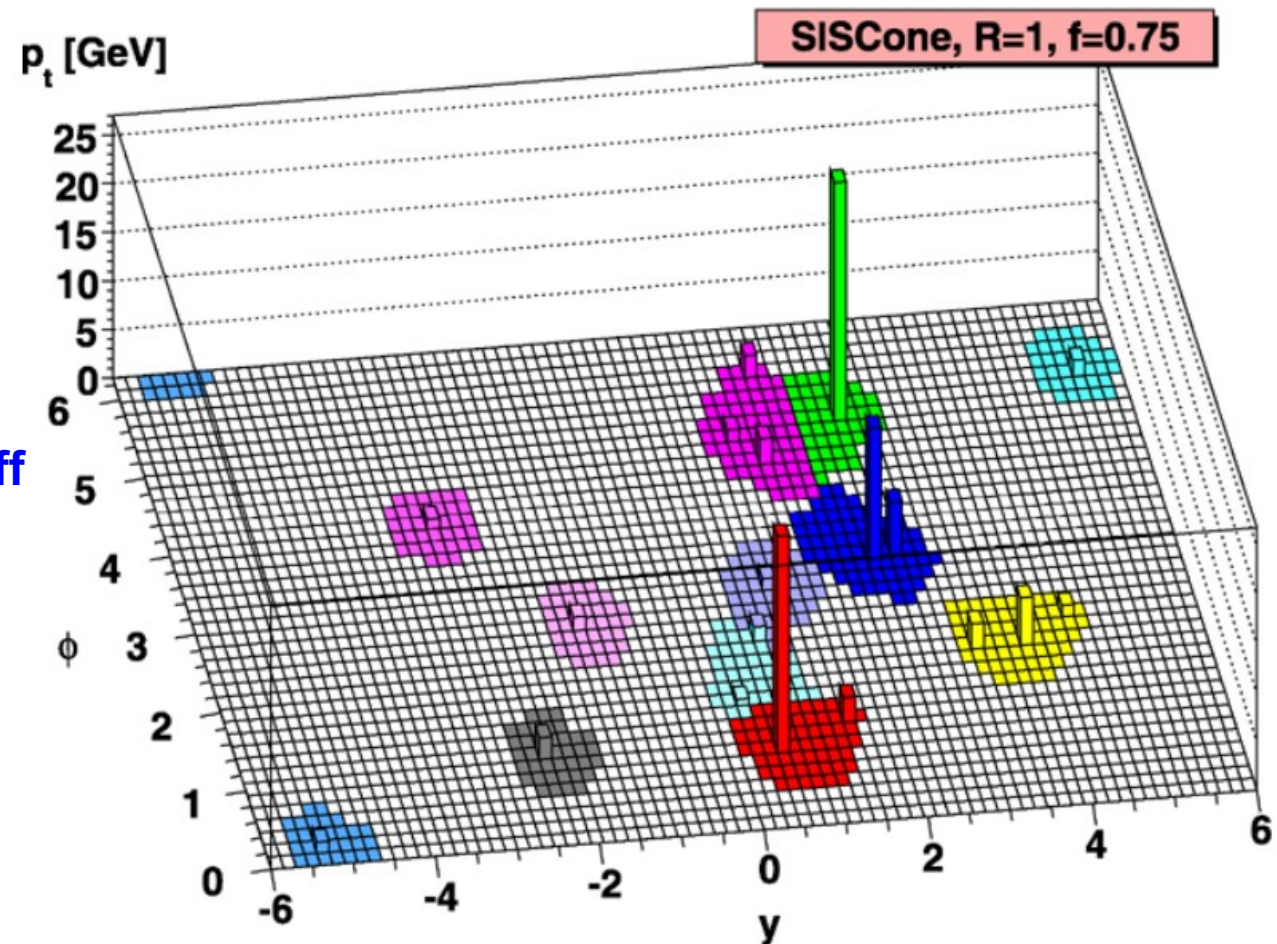
Cacciari, Salam, Soyez, JHEP 04 (2008) 063.



Character sheet

- ➔ True cone algorithm
- ➔ Does not use seeds (Seedless Infrared Safe)
- ➔ Cone-like shape in y, Φ plane
- ➔ Simpler to calibrate
- ➔ Smaller jet size
- ➔ Time per N particles $\sim N^2 \ln N$
- ➔ Used occasionally, never took off because of anti- k_T

e.g.: ZEUS, PLB 691 (2010) 127.



Salam, Soyez, JHEP 05 (2007) 086.



Jet algorithms for pp

Standard algorithm:

→ Anti- k_T :

ATLAS $R = 0.4, 0.6$

CMS $R = 0.5, 0.7$

(Run II: 0.4, 0.8)

→ k_T

→ SIScone (“real” cone algo)

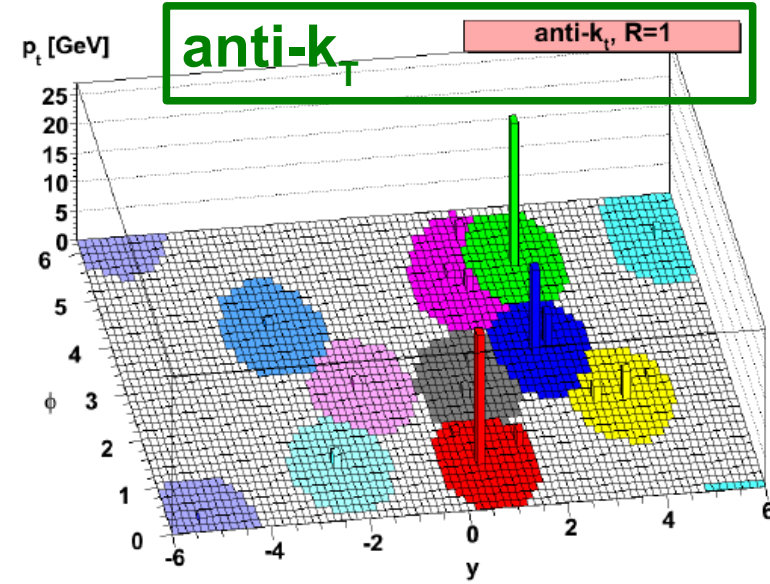
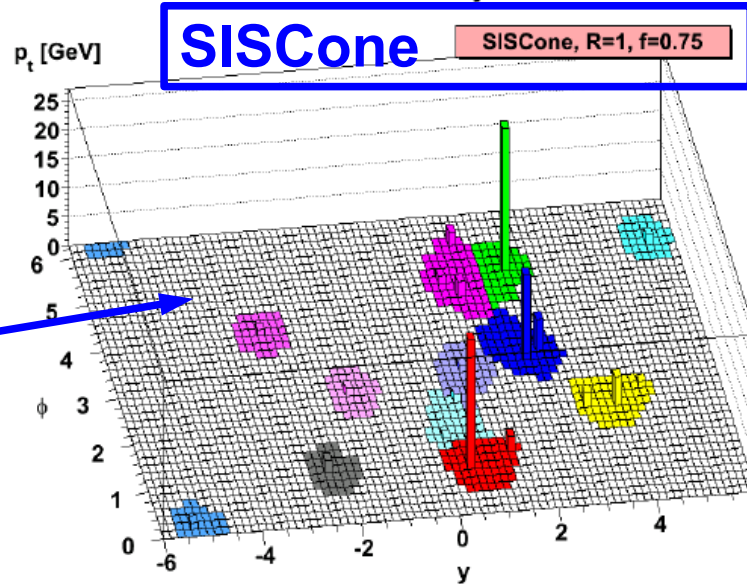
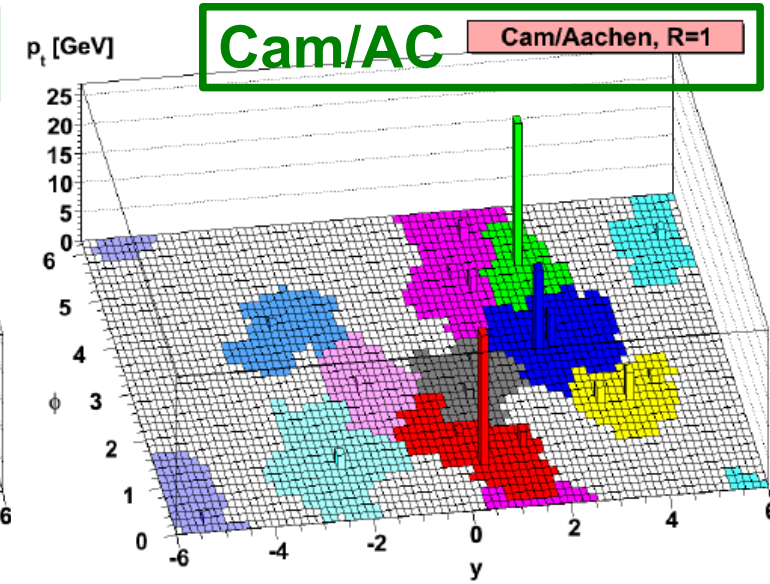
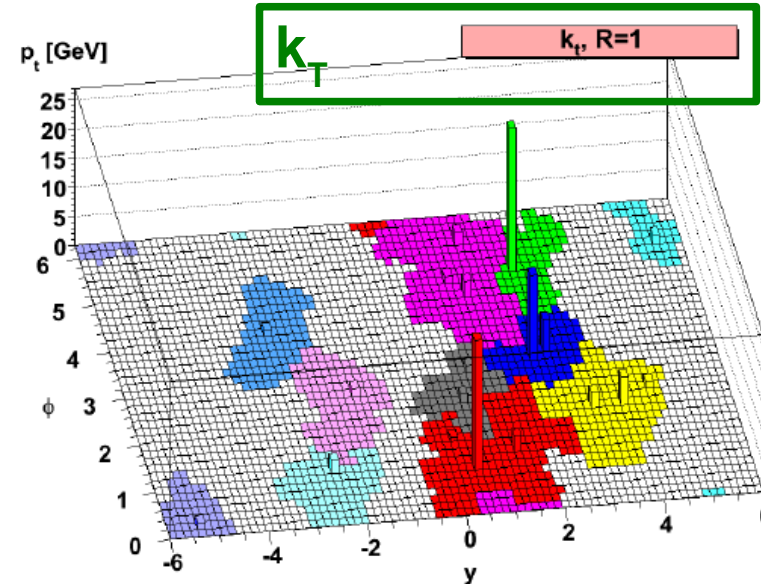
→ Cambridge/Aachen

useful in jet substructure,
e.g. for “boosted” top, t' , Z'

Often:

anti- k_T as baseline &
CamAC for substructure

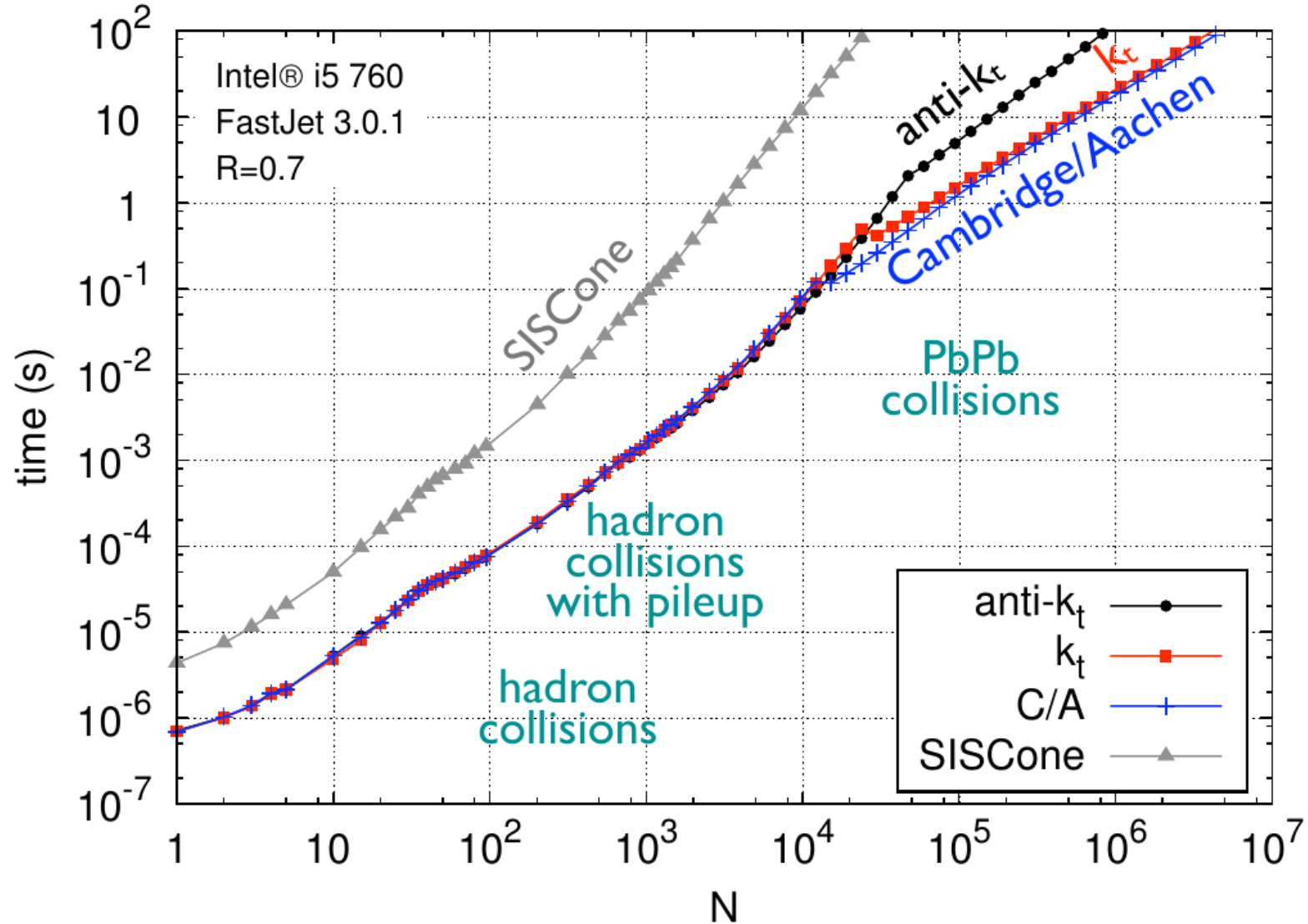
Only “real” cone
algorithm!





Timing comparison

Time to reconstruct an event with N particles





• Jet Algorithm Desiderata (Theory):

- ➔ **Infrared safety**
 - ➔ **Collinear safety**
 - ➔ **Longitudinal boost invariance**
(recombination scheme!)
 - ➔ **Boundary stability**
(→ 4-vector addition, rapidity y)
 - ➔ **Order independence**
(parton, particle, detector)
 - ➔ **Ease of implementation**
(standardized public code?)
- never problem for e^+e^- or ep
 - long-standing issue at Tevatron
 - solved for LHC: k_T family of jets

 - both solved with k_T family of jets

 - solved thanks to fastjet package

Cacciari et al., EPJC72 (2012) 1896.

See also:

“Snowmass Accord”, FNAL-C-90-249-E

Tevatron Run II Jet Physics, hep-ex/0005012

Les Houches 2007 Tools and Jets Summary , arXiv:0803.0678



• Jet Algorithm Desiderata (Experiment):

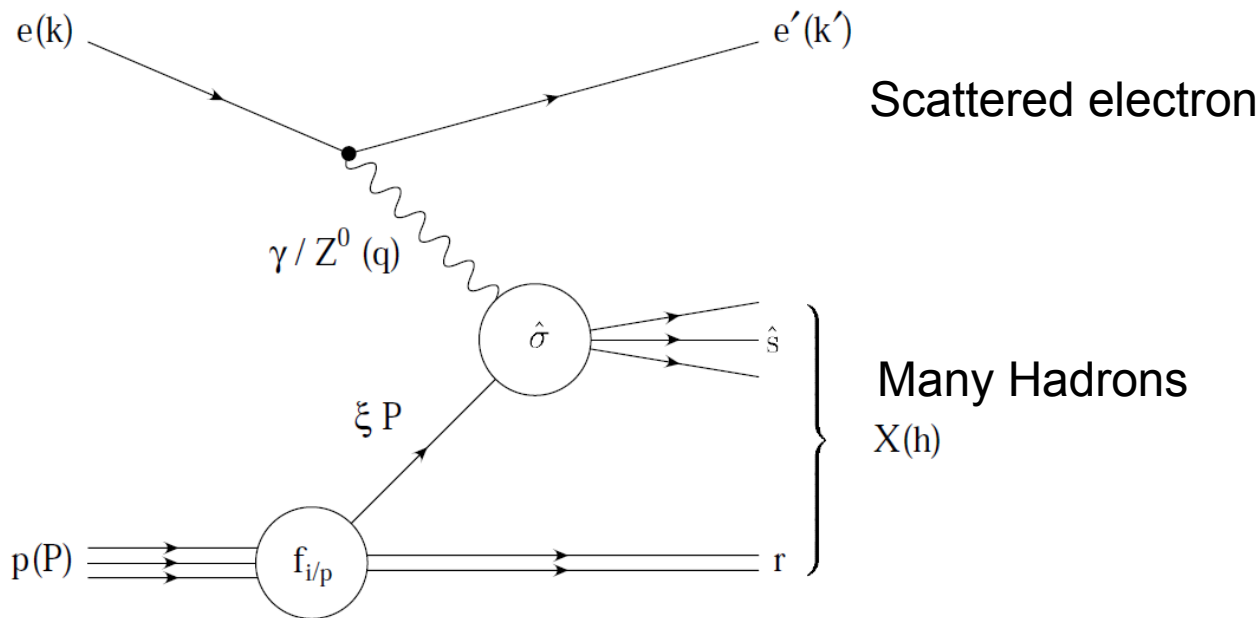
- ➔ **Computational efficiency and predictability**
(use in trigger?, reconstruction times?) - mostly solved thanks to fastjet
- ➔ **Maximal reconstruction efficiency**
(no dark jets) - solved with k_T family of jets
- ➔ **Minimal resolution smearing and angular biasing** - mostly solved by modern jet algorithms and unfolding
- ➔ **Insensitivity to pile-up**
(mult. collisions at high luminosity ...) - no unique answer, question of required jet radius
- ➔ **Ease of calibration** - solved by anti- k_T "cone" jets
- ➔ **Detector independence**
- ➔ **Fully specified (no erroneous reimplementations)** (details?, code?) - both solved thanks to fastjet package
- ➔ **Ease of implementation**
(standardized public code?)



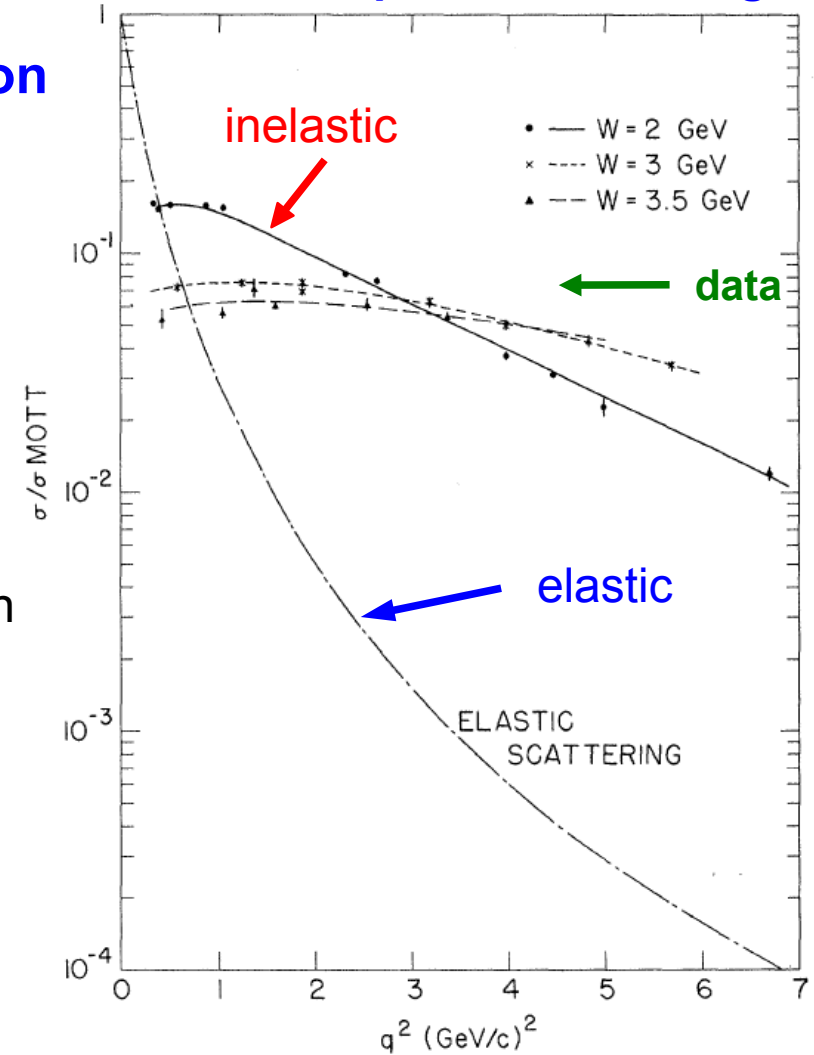
Scale invariance

- Inelastic \gg elastic cross section
- Inelastic cross section \sim const. * Mott x section
 - ➔ approximately independent of resolution $\sim q^2$
 - ➔ scale invariant, i.e. no natural length scale
 - ➔ like scattering at point-like objects

Deep-inelastic scattering (DIS)



electron-proton scattering



PRL 23 (1969) 935.



DIS kinematics

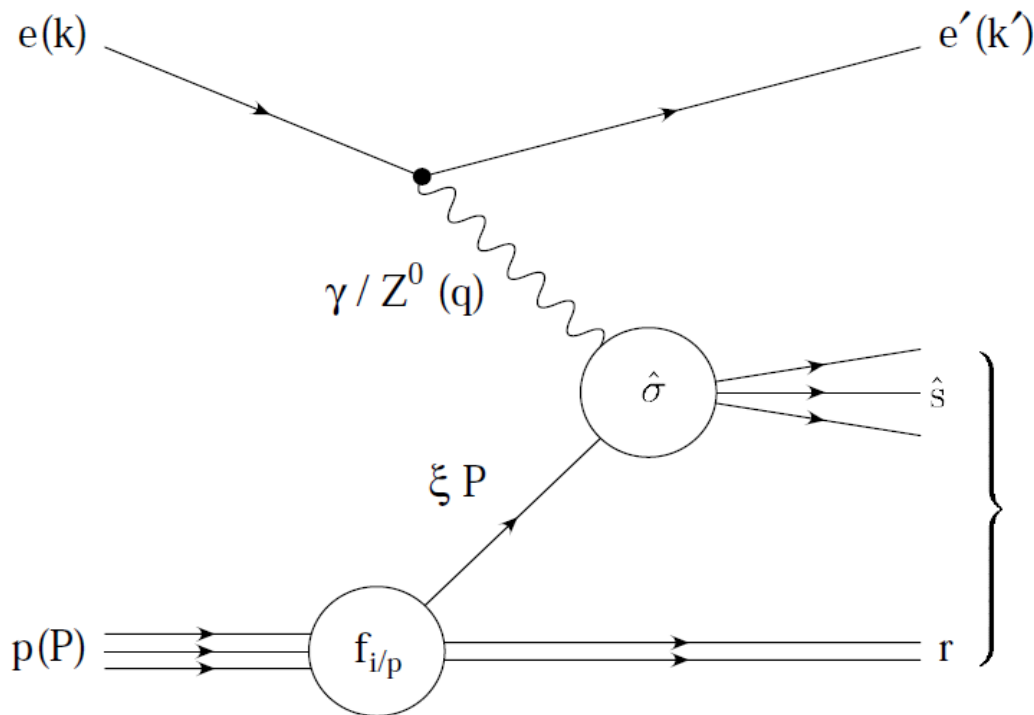
Neglect electron and proton masses: $M_p = M_e = 0$

Center-of-mass energy: $s = (k + P)^2 = 2k \cdot P = 4E_e E_p$

Elastic \rightarrow one independent variable:

$$Q^2 = -q^2 = (k - k')^2 = 2k \cdot k' = 2E_e E_{e'} (1 + \cos(\theta_{e'}))$$

Deep-inelastic scattering (DIS)



Inelastic \rightarrow 2nd variable:

$$W^2 = (q + P)^2 = 2P \cdot q - q^2$$

Alternative:

Bjorken scaling variable: $x = \frac{Q^2}{2P \cdot q}$

Inelasticity: $y = \frac{P \cdot q}{P \cdot k}$

$$0 \leq (x, y) \leq 1$$



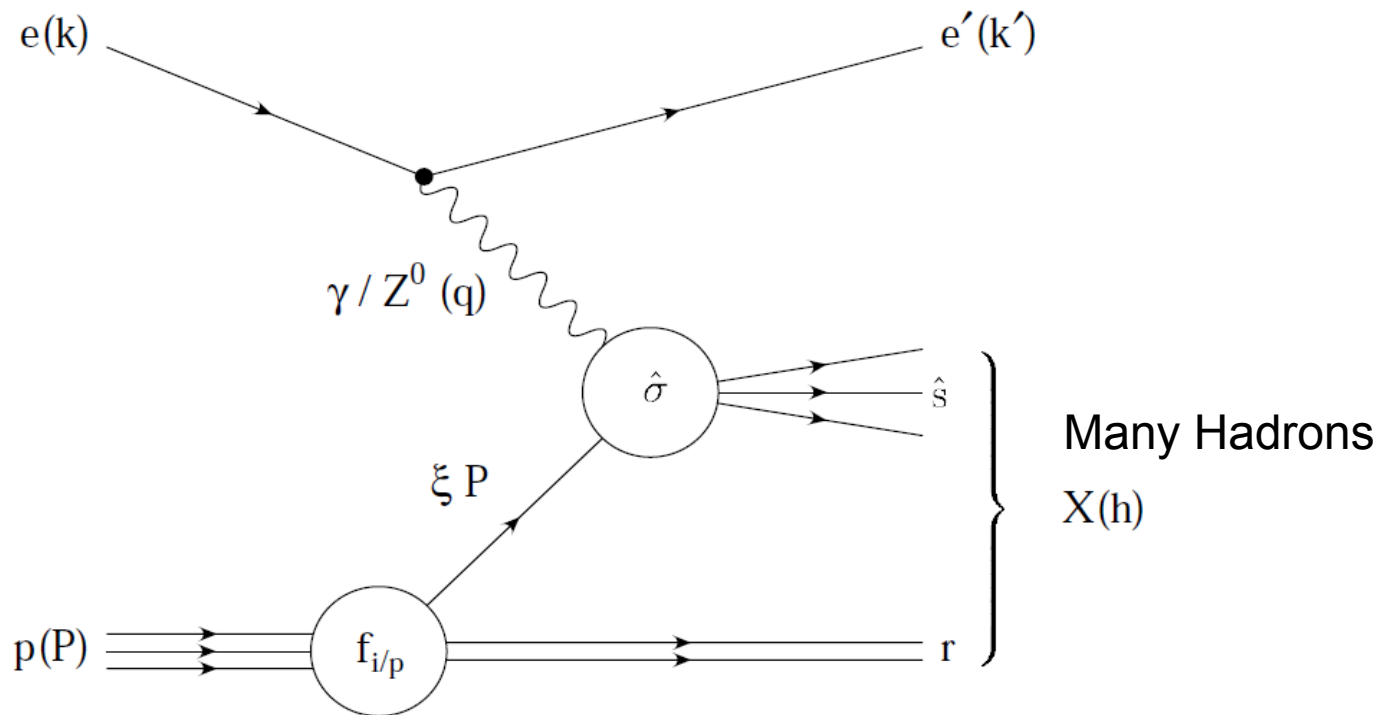
Infinite momentum frame



Proton equals collinear stream of fast moving “partons”, masses negligible
→ factorised incoherent partonic scatter

$$\hat{s} = (q + \xi P)^2 = 2\xi q \cdot P - Q^2 = \left(\frac{\xi}{x} - 1 \right) Q^2$$

Deep-inelastic scattering (DIS)



$$x = \frac{Q^2}{2P \cdot q}$$

$$y = \frac{P \cdot q}{P \cdot k}$$

$$0 \leq (x, y) \leq 1$$

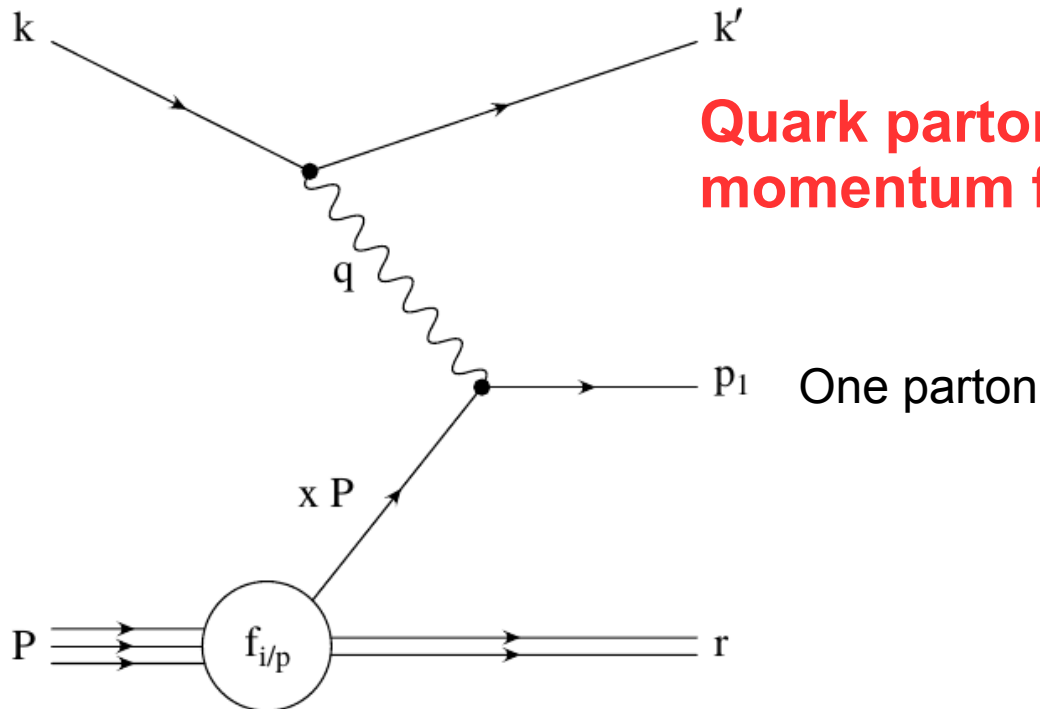


Infinite momentum frame

Proton equals collinear stream of fast moving “partons”, masses negligible
→ factorised incoherent partonic scatter

$$\hat{s} = (q + \xi P)^2 = 2\xi q \cdot P - Q^2 = \left(\frac{\xi}{x} - 1 \right) Q^2$$

Deep-inelastic scattering (DIS) at LO $\hat{s} = 0 \rightarrow \xi = x$ $\xi = \left(1 + \frac{\hat{s}}{Q^2} \right) x$



Quark parton model (QPM) → scaling variable x: momentum fraction of struck parton in proton

$$x = \frac{Q^2}{2P \cdot q}$$

$$y = \frac{P \cdot q}{P \cdot k}$$

$$0 \leq (x, y) \leq 1$$



Infinite momentum frame



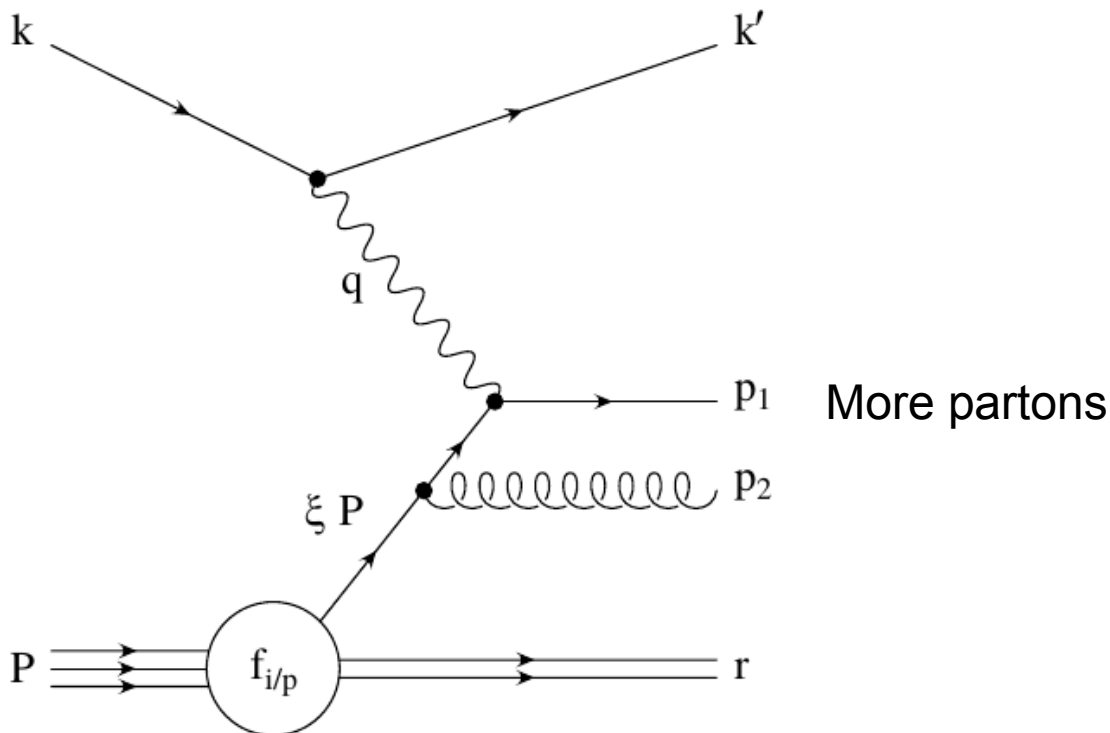
Proton equals collinear stream of fast moving “partons”, masses negligible
→ factorised incoherent partonic scatter

$$\hat{s} = (q + \xi P)^2 = 2\xi q \cdot P - Q^2 = \left(\frac{\xi}{x} - 1\right) Q^2$$

Deep-inelastic scattering (DIS) at NLO

$$x \leq \xi \leq 1$$

$$\xi = \left(1 + \frac{\hat{s}}{Q^2}\right) x$$



$$x = \frac{Q^2}{2P \cdot q}$$

$$y = \frac{P \cdot q}{P \cdot k}$$

$$0 \leq (x, y) \leq 1$$



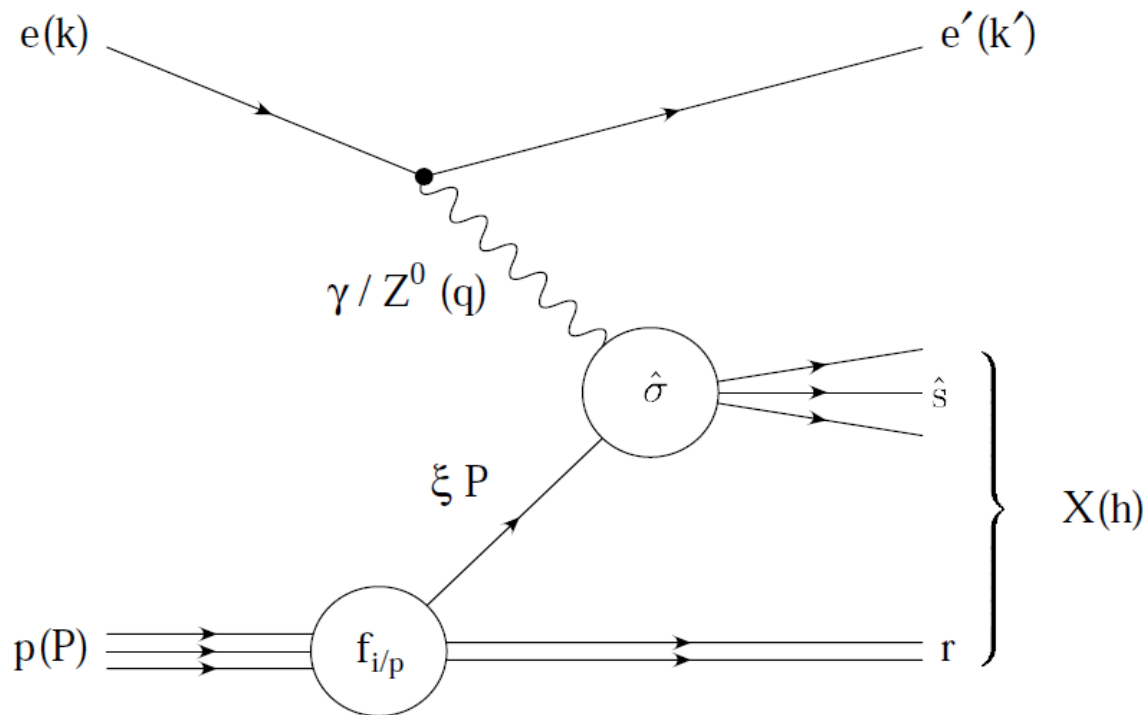
Target rest frame

Inelasticity y is relative energy loss of electron in target rest frame:

$$y = \frac{E_e - E_{e'}}{E_e}$$

Only two of the four invariant variables are independent!

Deep-inelastic scattering (DIS)



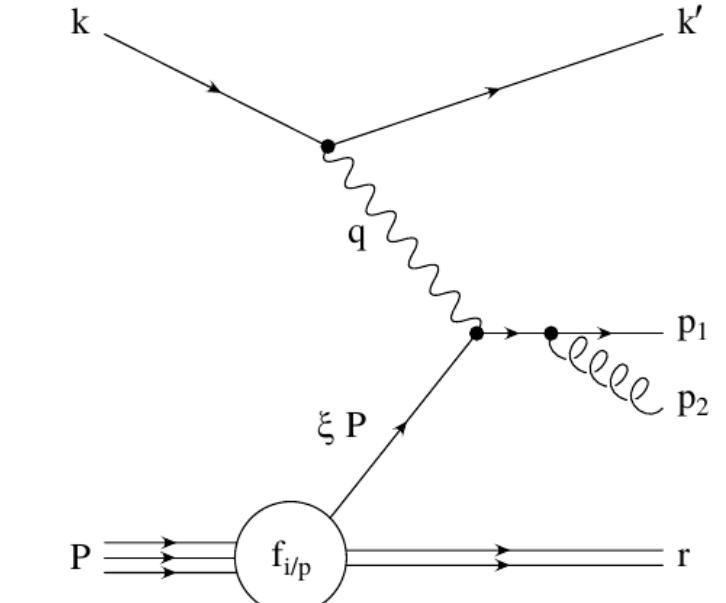
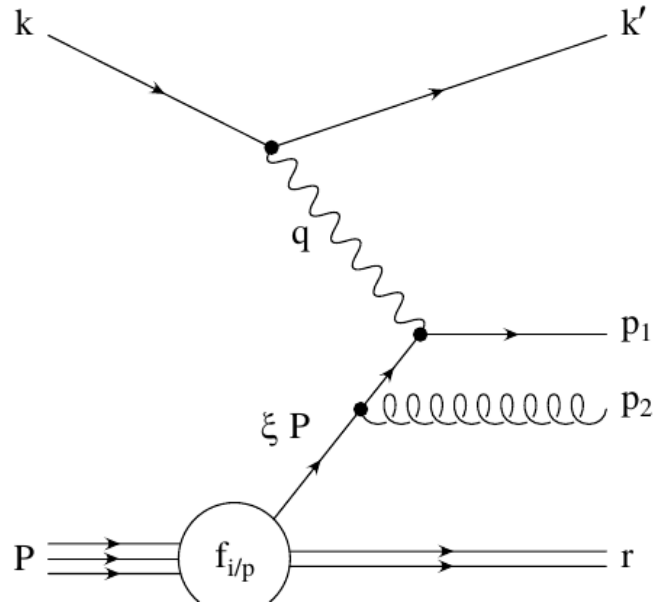
Conversion formulae

$$x = \frac{Q^2}{Q^2 + W^2} \quad y = \frac{Q^2 + W^2}{s}$$

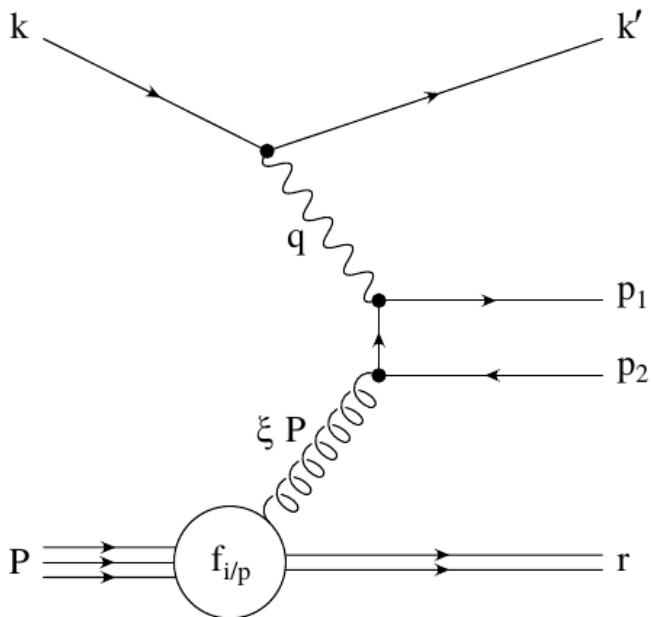
$$Q^2 = sxy \quad W^2 = s(1 - x)y$$



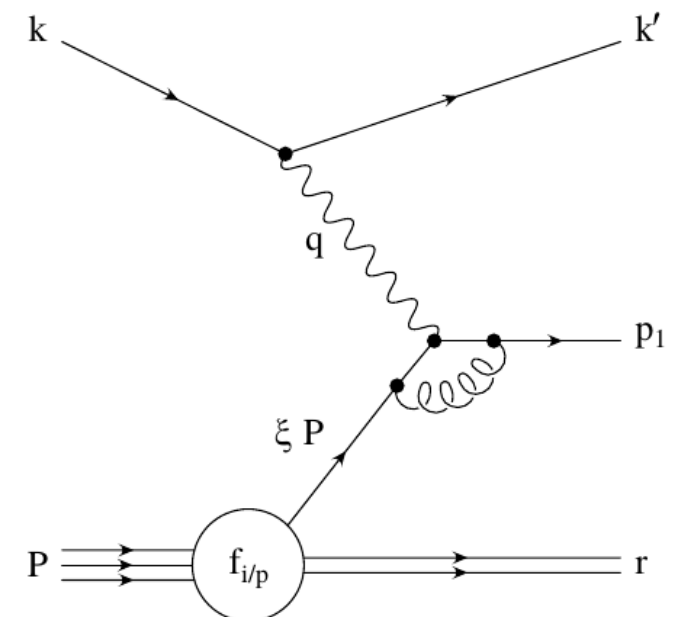
QCD-Compton



Boson-gluon fusion



Virtual correction





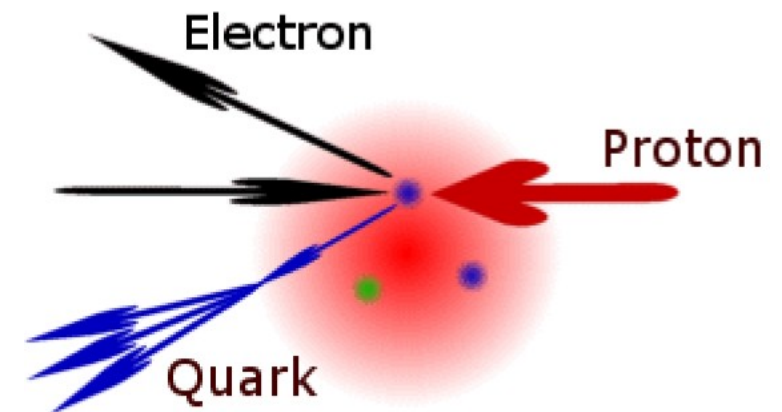
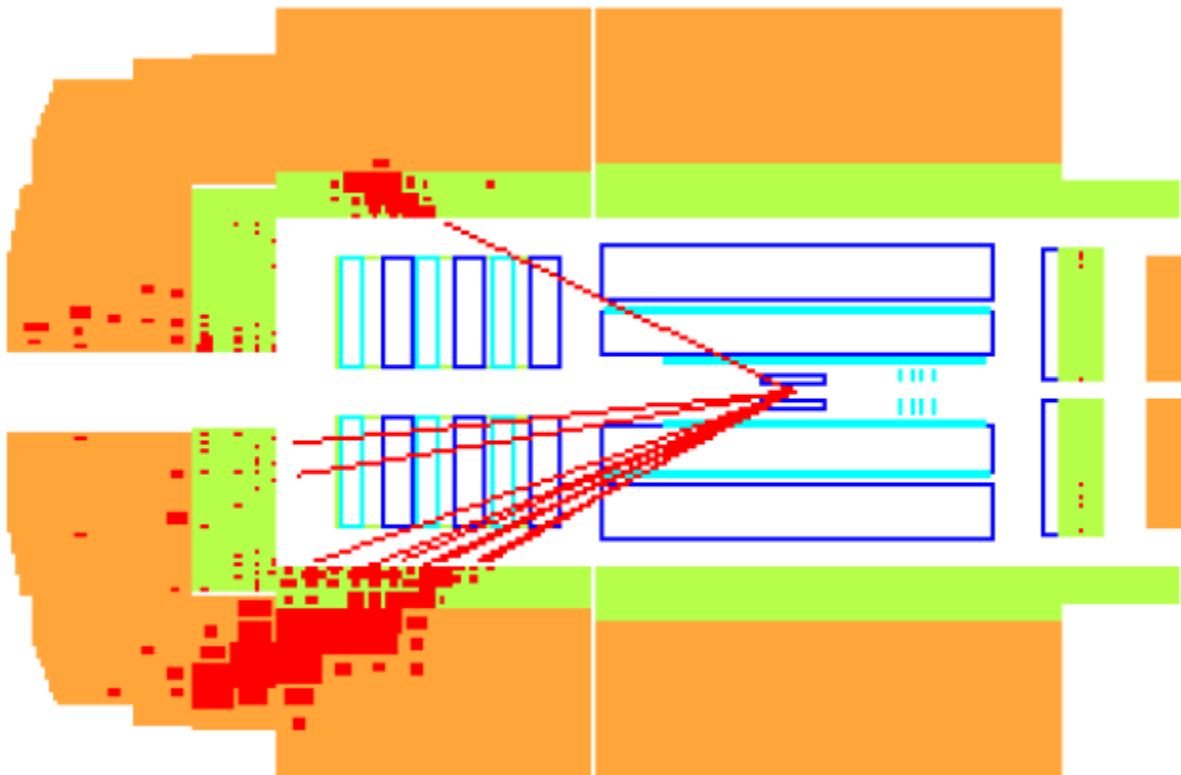
Deep-inelastic scattering at HERA



Electromagnetic reaction:

Backscattering of electron off charged proton constituent

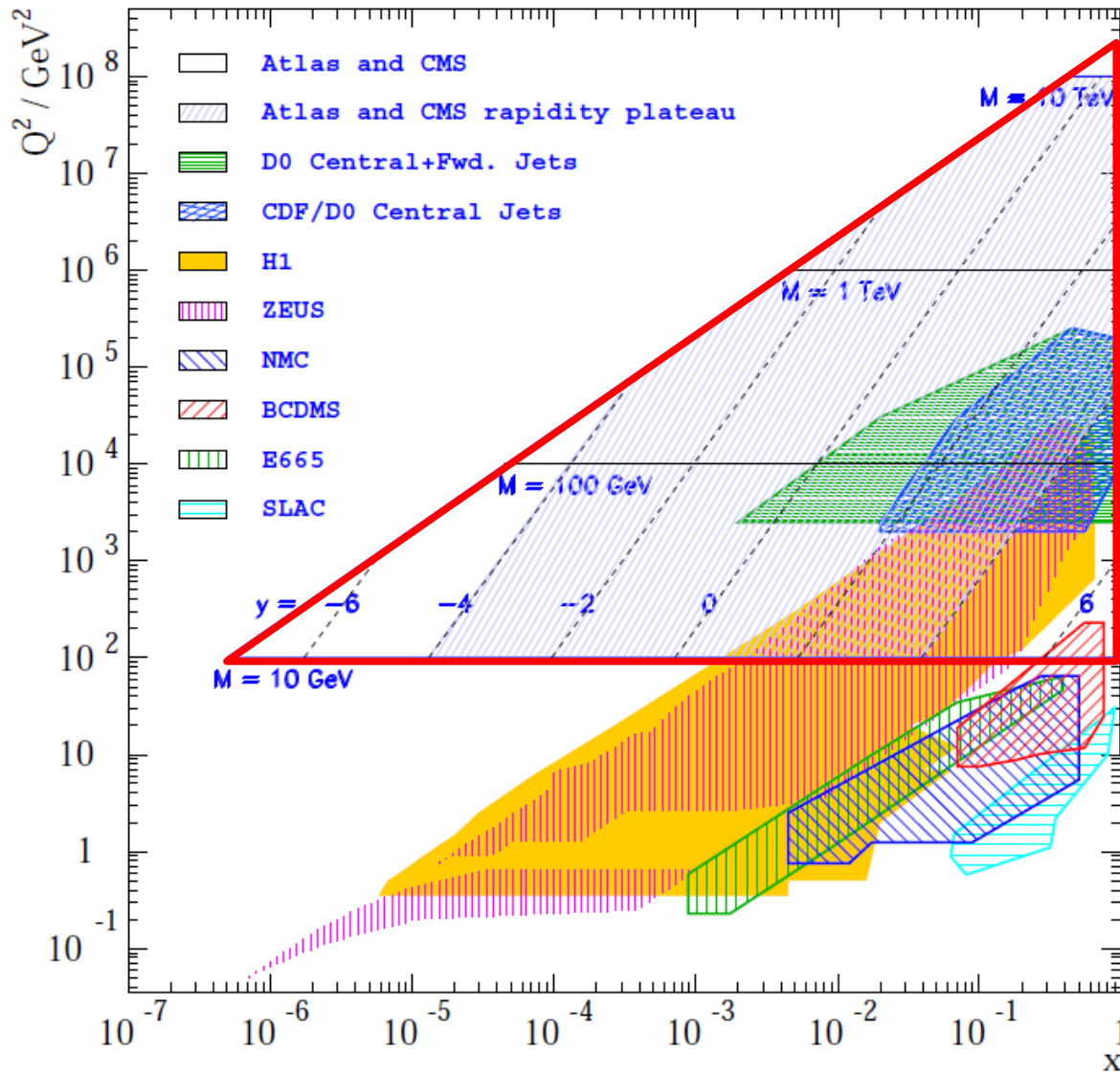
H1 Detector



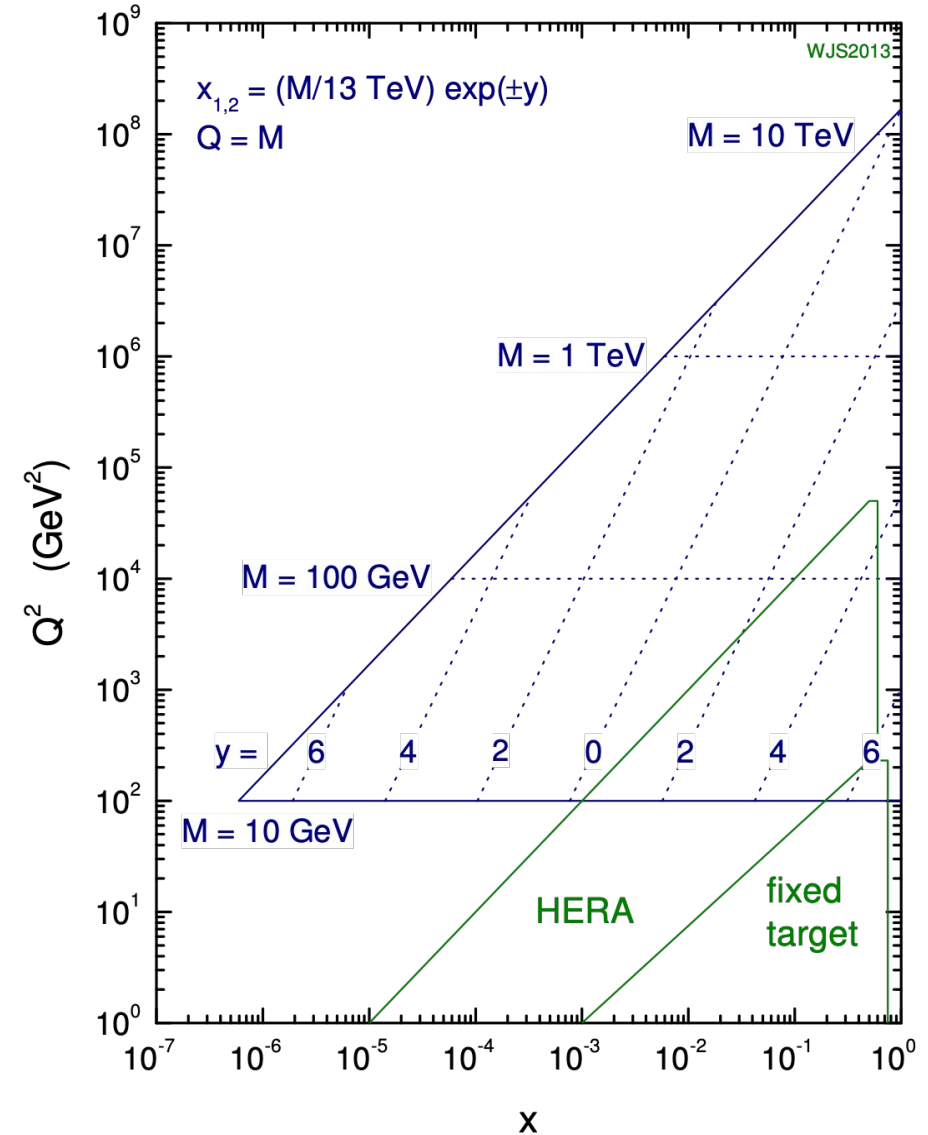
H1 Event Tutorial, J Meyer, DESY (2005)



Phase space in x and Q^2



13 TeV LHC parton kinematics



S. Glazov, Braz.J.Ph. 37 (2007) 793.

W.J. Stirling



$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[(1-y) \frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2) \right] \quad Q^2 \gg M_p^2 y^2$$

$F_1(x, Q^2)$ and $F_2(x, Q^2)$ are structure functions incorporating the form factors (and kinematic ones, τ), but cannot be related to Fourier transforms any more since dependent on x .

Still, $F_1(x, Q^2)$ is of purely magnetic origin, while $F_2(x, Q^2)$ originates from both, electric and magnetic effects.

What do they mean?



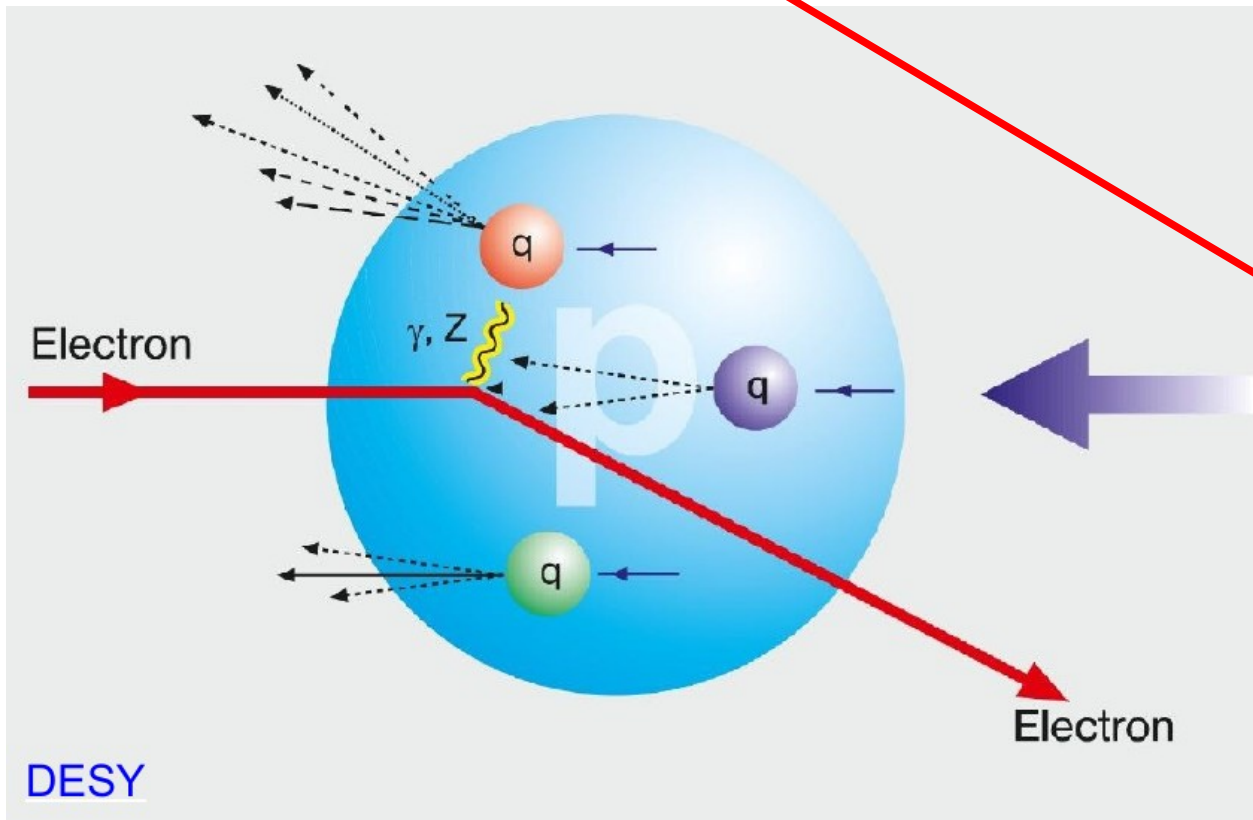
Quark-Parton-Model



J.D. Bjorken, R.P. Feynman 1969:

• Infinite momentum frame

- ➔ incoherent superposition of elastic scatterings with point-like “partons”
- ➔ scale invariant, i.e. independent of resolution $\sim q^2$, no natural length scale
- ➔ partons have spin 1/2



Bjorken scaling:

$$F_1(x, Q^2) \rightarrow F_1(x)$$

$$F_2(x, Q^2) \rightarrow F_2(x)$$

Callan-Gross relation:

$$F_1(x) = \frac{F_2(x)}{2x}$$

Spin 0 would give:

$$F_1(x) = 0$$



Quark-Parton-Model

Modern writing:

$$F_2(x) = x \sum_i e_i^2 [q_i(x) + \bar{q}(x)]$$

← quark charges
← anti-quark momentum distribution

← quark momentum distribution

q_i : parton distribution functions (PDFs)

Bjorken scaling:

$$F_1(x, Q^2) \rightarrow F_1(x)$$

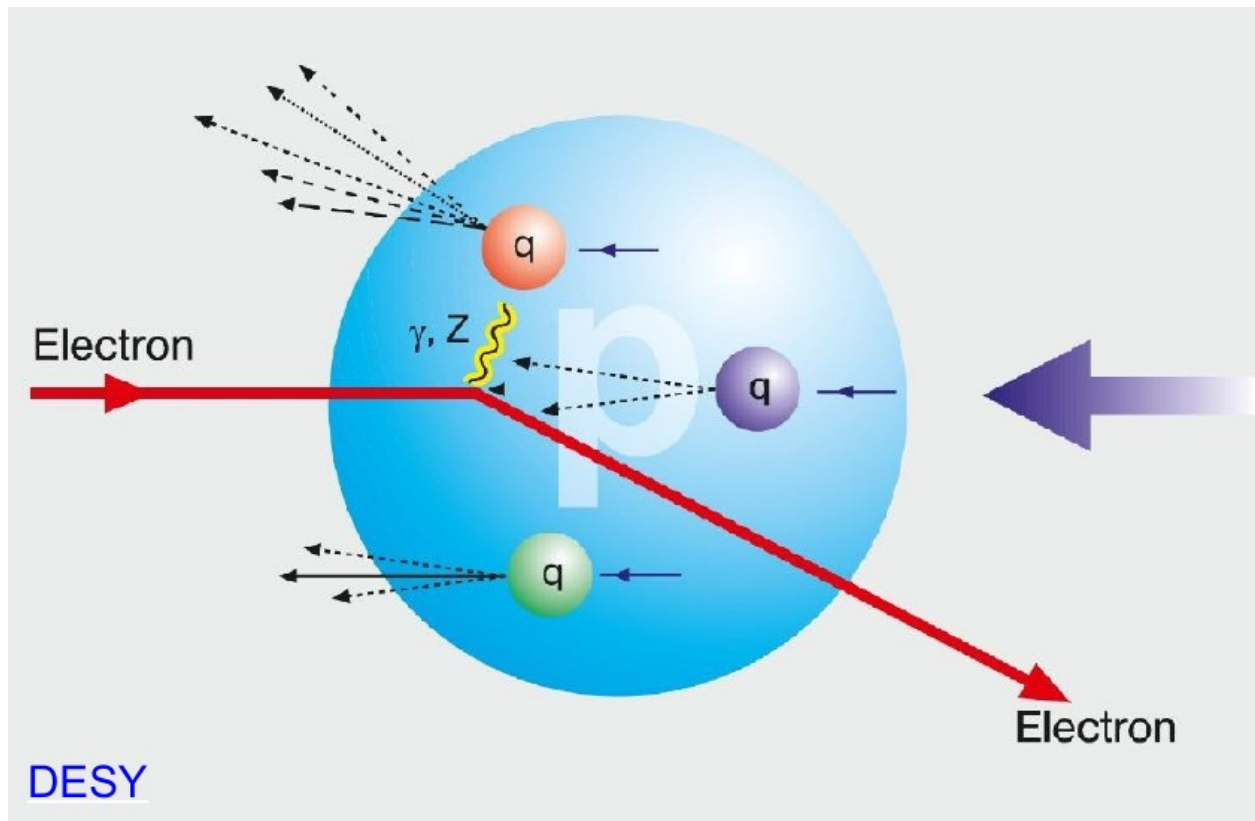
$$F_2(x, Q^2) \rightarrow F_2(x)$$

Callan-Gross relation:

$$F_1(x) = \frac{F_2(x)}{2x}$$

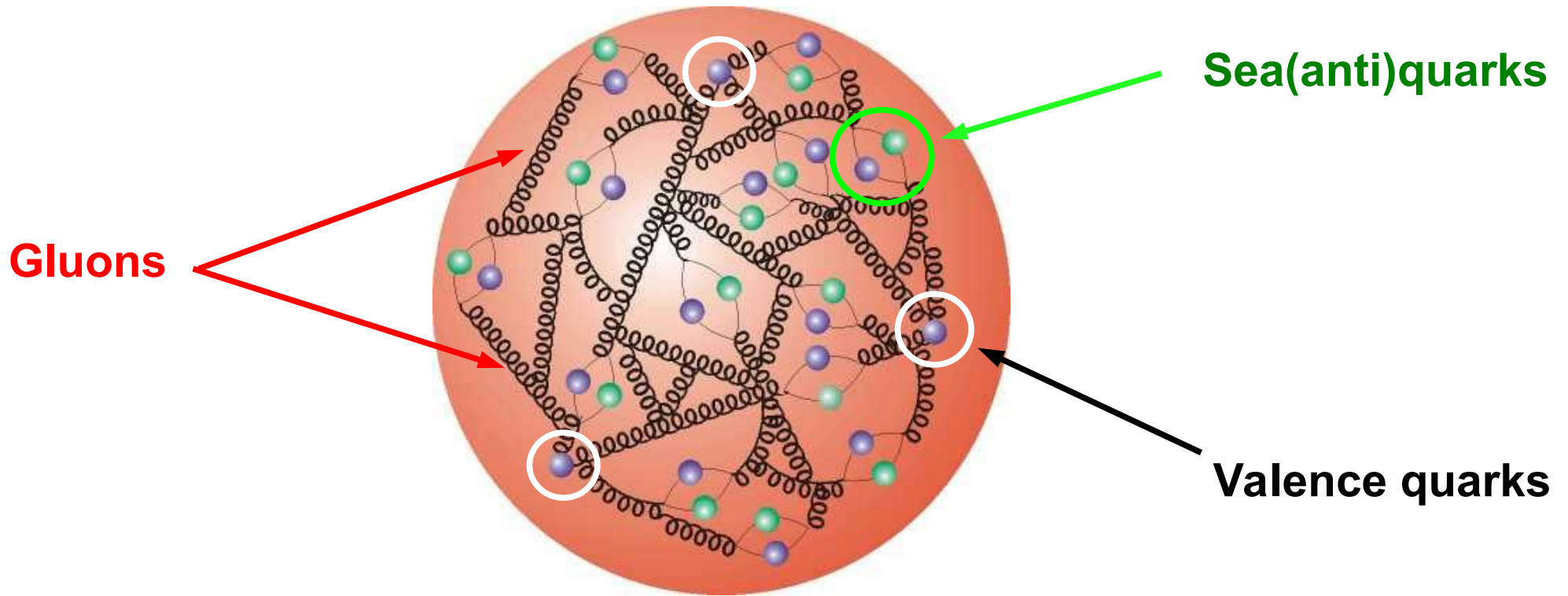
Spin 0 would give:

$$F_1(x) = 0$$





Proton structure



- Example of generic functional form of proton structure:

$$x f(x) = Ax^B (1 - x)^C (1 + Dx + Ex^2)$$

Normalisation

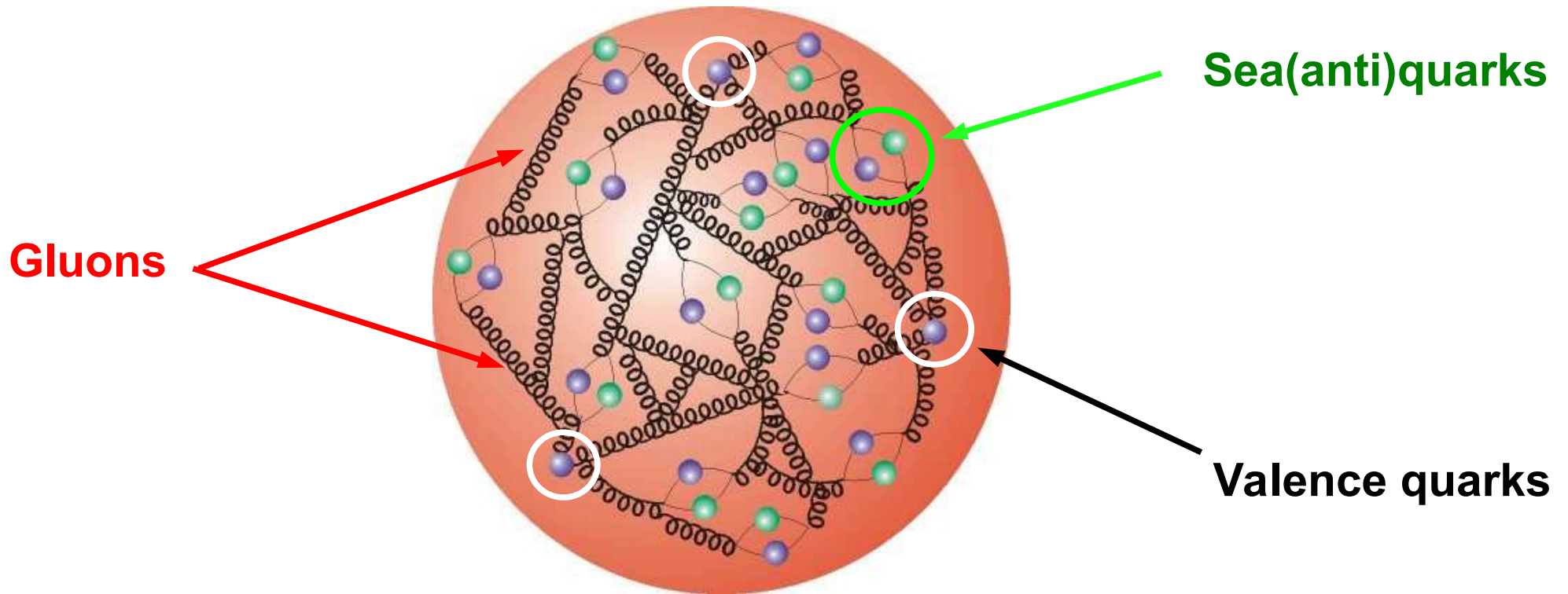
Behaviour for $x \rightarrow 0$

Behaviour for $x \rightarrow 1$

Middle region
largest variability



Proton structure



$$xf(x) = Ax^B(1-x)^C(1+Dx+Ex^2)$$

- non-perturbative in nature
- allow description of hadron collisions
- x-dependence extracted from measurements, e.g. DIS
- also depend on factorisation scale μ_F
- μ_F dependence given by perturbative QCD (DGLAP)



Proton structure à la HERAPDF



- Typical parametrisation of the proton structure:

$$xf(x) = Ax^B(1-x)^C(1+Dx+Ex^2)$$

Normalisation

Behaviour for $x \rightarrow 0$

Behaviour for $x \rightarrow 1$

Middle region
largest variability

- And this for all flavours ... here an example from HERAPDF:

$$xg(x) = A_g x^{B_g} (1-x)^{C_g} - A'_g x^{B'_g} (1-x)^{C'_g}$$

Gluon

$$xu_v(x) = A_{u_v} x^{B_{u_v}} (1-x)^{C_{u_v}} (1 + D_{u_v} x + E_{u_v} x^2)$$

Valence
quarks

$$q_v = q - \bar{q}$$

$$xd_v(x) = A_{d_v} x^{B_{d_v}} (1-x)^{C_{d_v}}$$

only while
 $\Delta\sigma$ calc.

Sea
quarks

$$x\bar{U}(x) = A_{\bar{U}} x^{B_{\bar{U}}} (1-x)^{C_{\bar{U}}}$$

$$x\bar{D}(x) = A_{\bar{D}} x^{B_{\bar{D}}} (1-x)^{C_{\bar{D}}}$$

→ 19 parameters

$$\bar{U} = \bar{u} \quad \bar{D} = \bar{d} + \bar{s}$$



Scale dependence given by pQCD



Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations:

Scale dep. μ_f^2

$$\mu_f^2 \frac{\partial f_i(x, \mu_f)}{\partial \mu_f^2} = \sum_{j=\{q, \bar{q}, g\}} \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P_{ij}(z) f_{j/p}(x/z, \mu_f)$$

PDFs

Momentum fractions z

LO Altarelli-Parisi (AP) splitting functions:

$$P_{qq}(z) = C_F \left(\frac{1+z^2}{(1-z)_+} + \frac{3}{2} \delta(1-z) \right)$$

quark \rightarrow quark splitting

$$P_{gq}(z) = C_F \left(\frac{1+(1-z)^2}{z} \right)$$

quark \rightarrow gluon splitting

$$P_{qg}(z) = T_F (z^2 + (1-z)^2)$$

gluon \rightarrow quark splitting

$$P_{gg}(z) = 2C_A \left(\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z) \right) + \delta(1-z) \frac{11C_A - 4N_F T_F}{6}$$

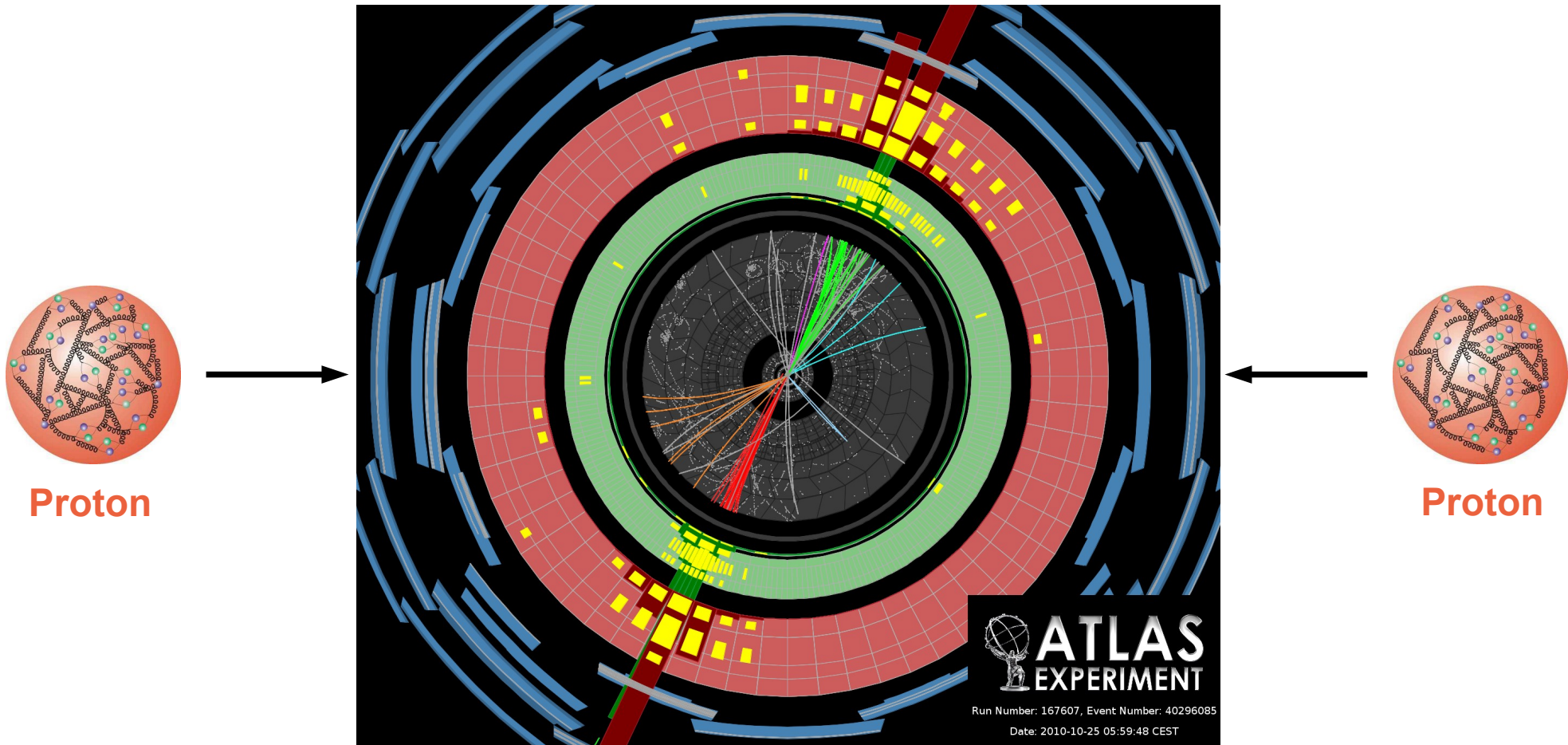
gluon \rightarrow gluon splitting

QCD SU(3)



Hadron-hadron collisions

“Broadband beams” of various parton types with various energies
→ QCD parton collider!



Challenge: Reliable calculations of observables

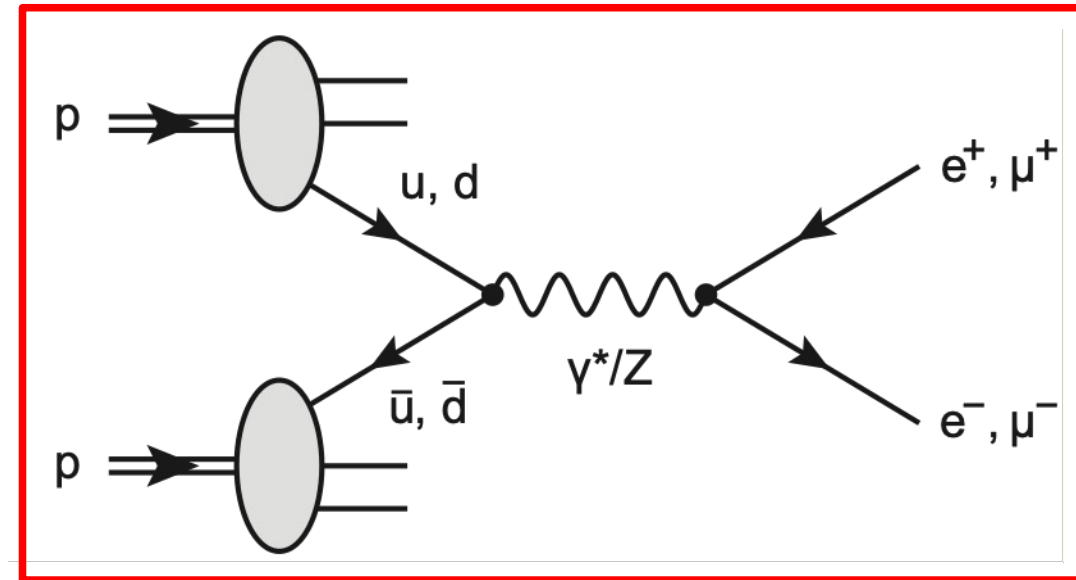


Prototype process: Drell-Yan

$$pp \rightarrow l^+ l^- + X$$

• Hadro-production of lepton pairs

- at large center-of-mass energies
- with large invariant mass
- color-neutral final state (except proton remnants) \rightarrow no hadronisation



Not a Feynman diagram

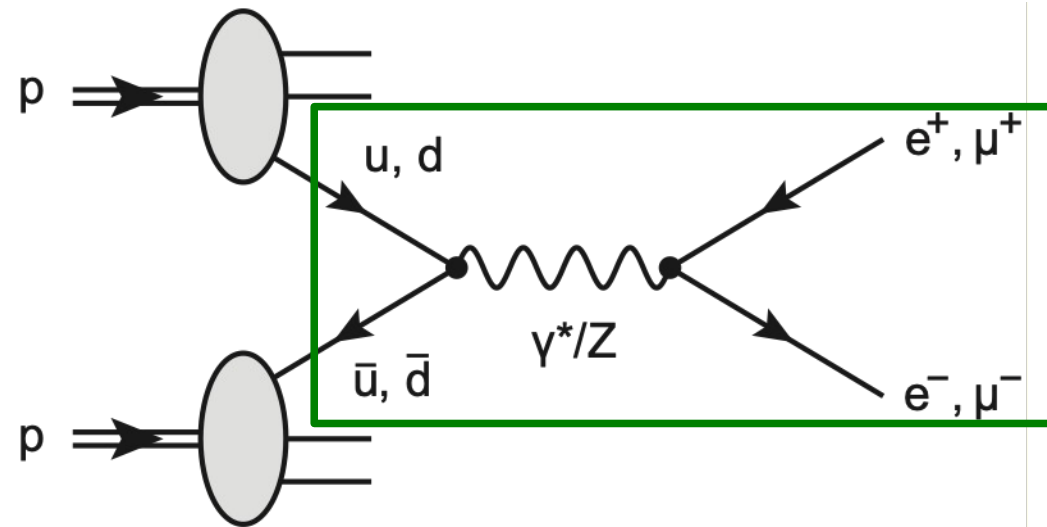


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$$pp \rightarrow l^+ l^- + X$$

• Hadro-production of lepton pairs

- at large center-of-mass energies
- with large invariant mass
- color-neutral final state (except proton remnants) \rightarrow no hadronisation



Partonic Feynman diagram
 \rightarrow **calculable in perturbative QCD**

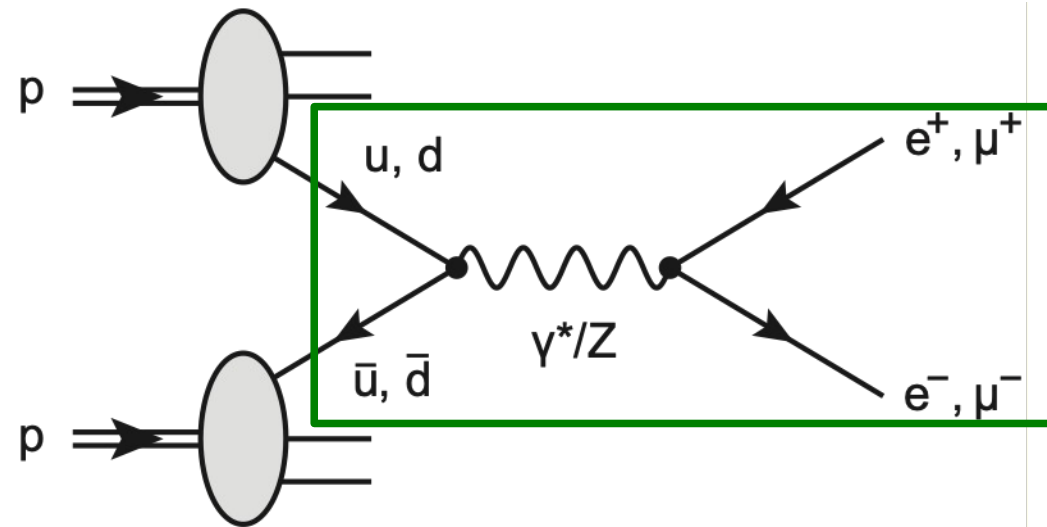


Prototype process: Drell-Yan

$$pp \rightarrow l^+ l^- + X$$

● Hadro-production of lepton pairs

- ➔ at large center-of-mass energies
- ➔ with large invariant mass
- ➔ color-neutral final state (except proton remnants) → no hadronisation



● Factorisation theorem of QCD:

- ➔ Process can be calculated by factorising “hard” and “soft” components
 - ➔ Calculate hard partonic subprocess
 - ➔ Weight cross section with probability to find partons with momenta x_1, x_2 inside hadrons
 - ➔ Integrate over all possible parton momenta
 - ➔ Sum over all possible parton flavors

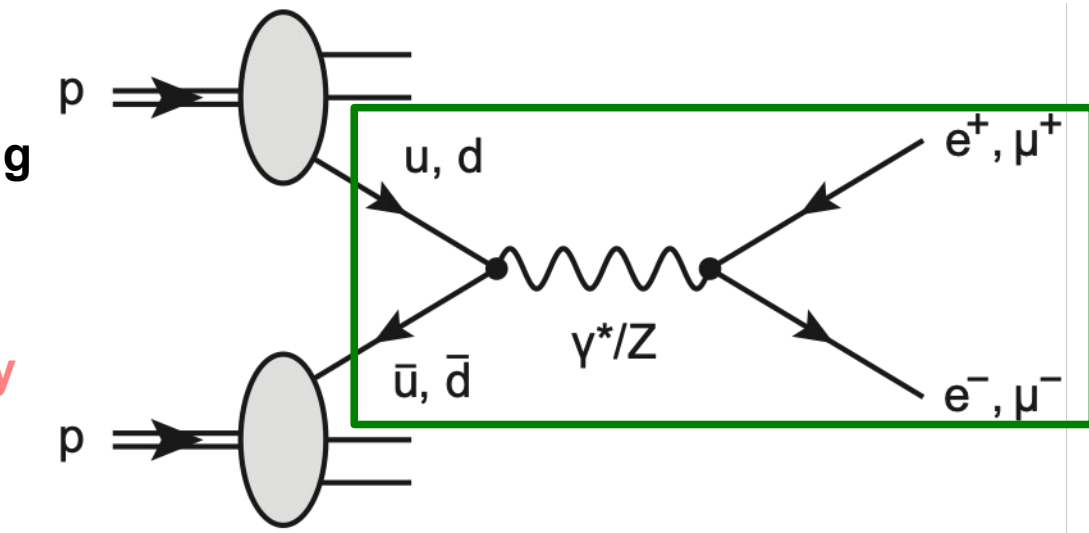


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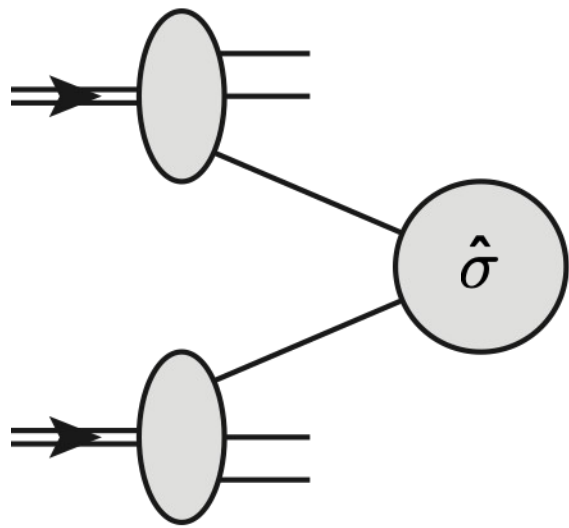


$$\sigma_{\text{DY}} = \sum_{i,j} \int dx_i dx_j f_i(x_i) f_j(x_j) \cdot \hat{\sigma}(q_i q_j \rightarrow l^+ l^-)$$

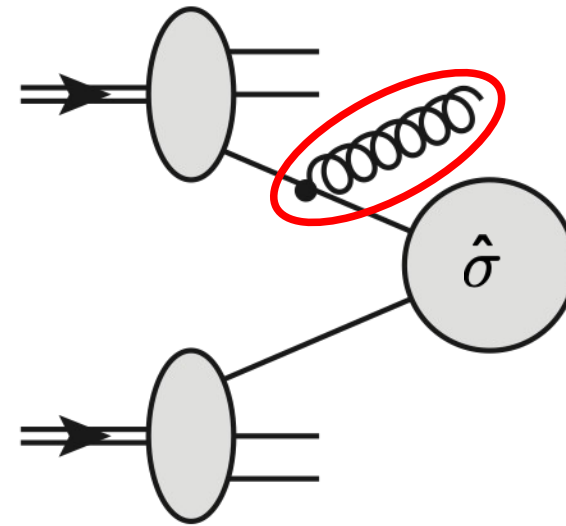
PDFs $f_i(x_i)$ are universal; can be measured independently e.g. in DIS!



Factorisation scale



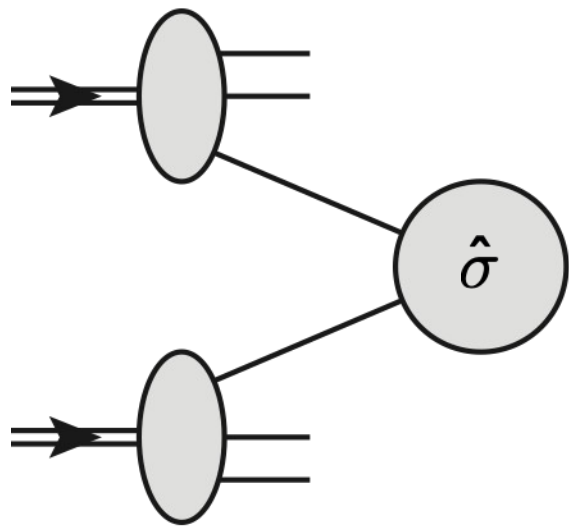
higher orders



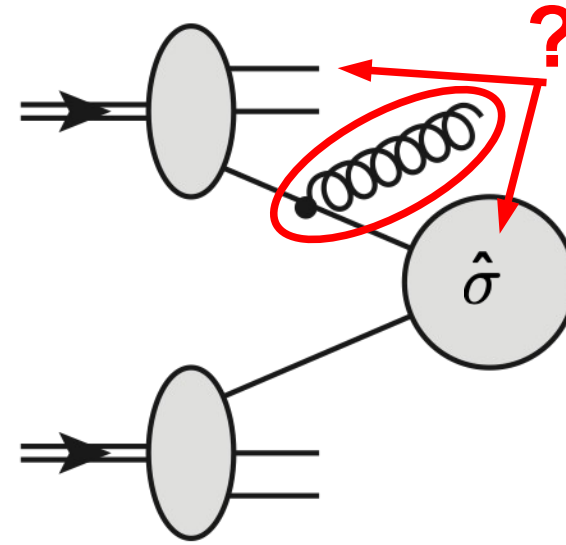
Where does this belong to?



Factorisation scale



higher orders



Where does this belong to?

The PDF?

Or the parton process?

Attribution ambiguous:

- ➔ Leads to soft and/or collinear divergences (long-distance effects!)
- ➔ Solution: Introduce a new scale to separate short- and long-distance effects
 - ➔ Factorisation scale μ_f
 - ➔ All soft and collinear divergences (long-distance effects) are absorbed into the PDFs determined from experimental measurements

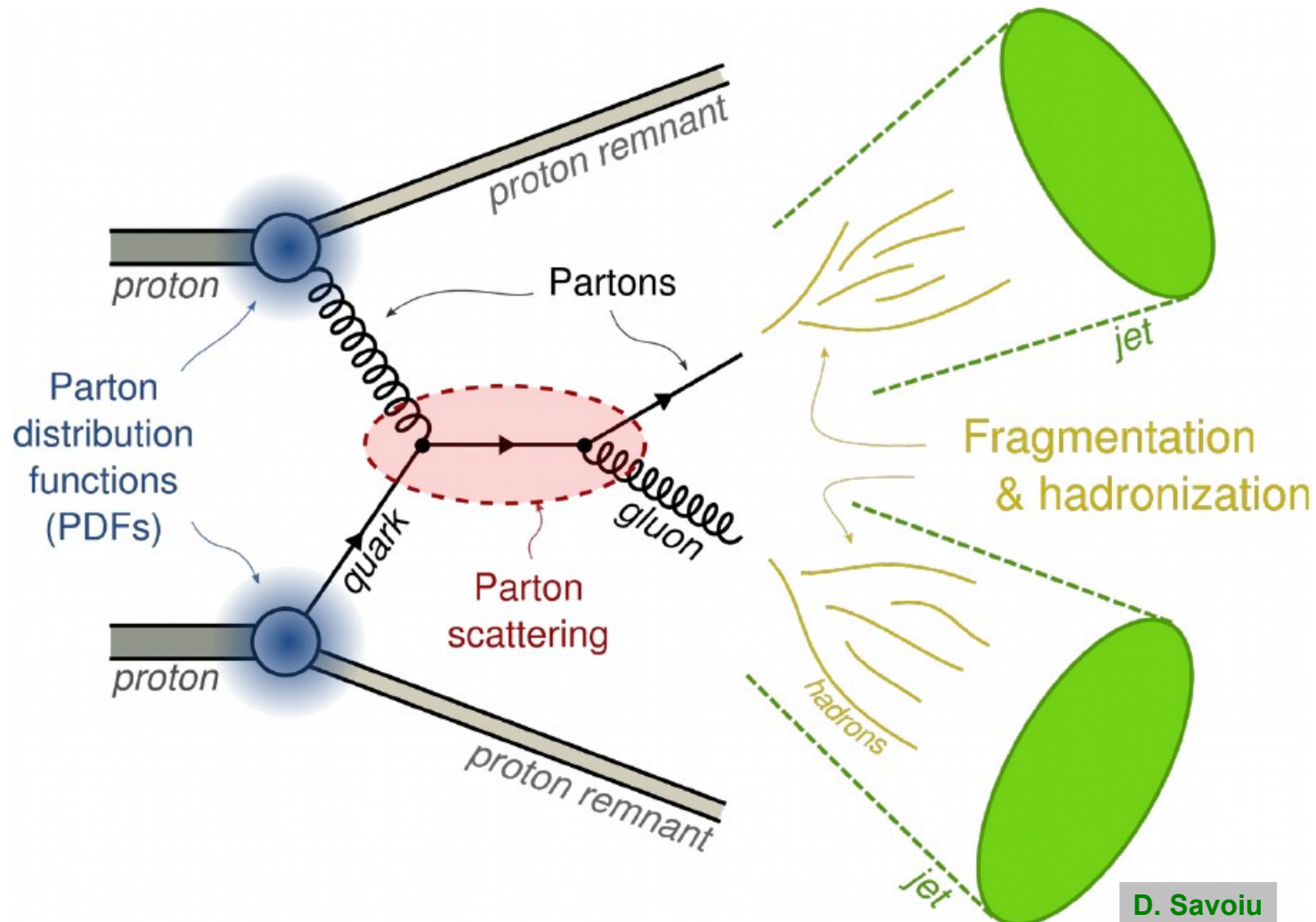
$$f_i(x_i) \rightarrow f_i(x_i, \mu_f^2)$$

$$\hat{\sigma}_{ij}(x_i, x_j, \mu_r^2, \alpha_s(\mu_r^2)) \rightarrow \hat{\sigma}_{ij}(x_i, x_j, \mu_f^2, \mu_r^2, \alpha_s(\mu_r^2))$$



Hadron-hadron cross sections

Factorisation successful also for more general final states, e.g. jet production!



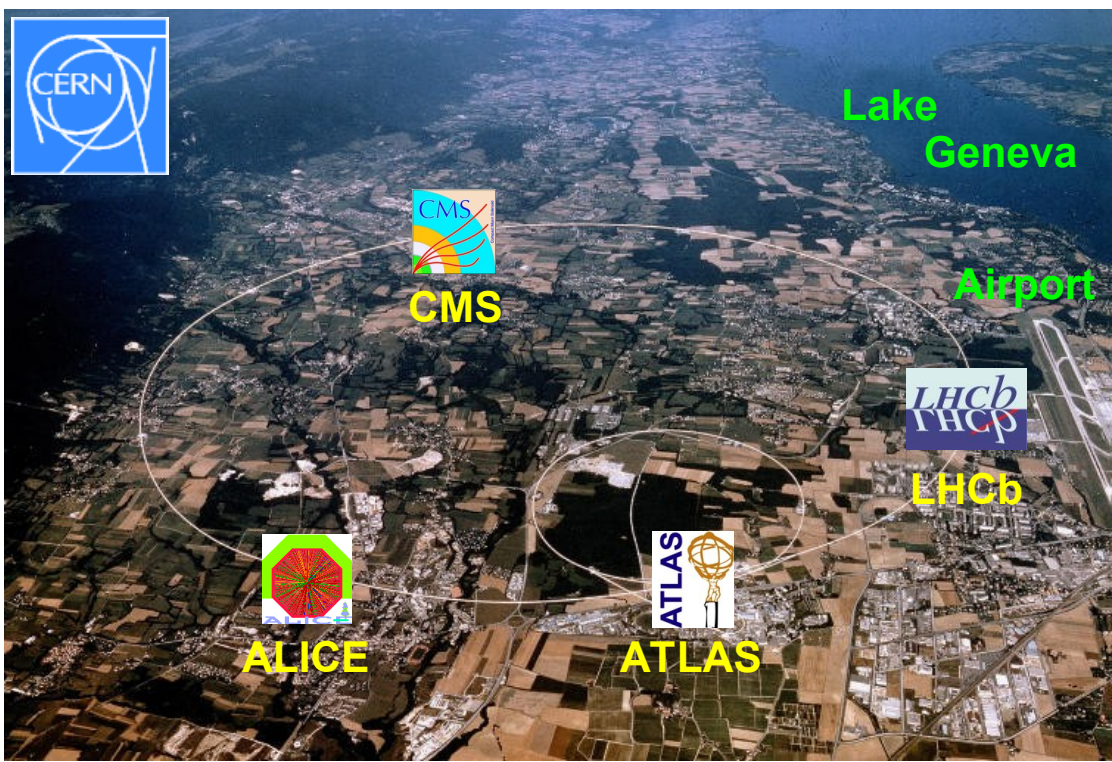
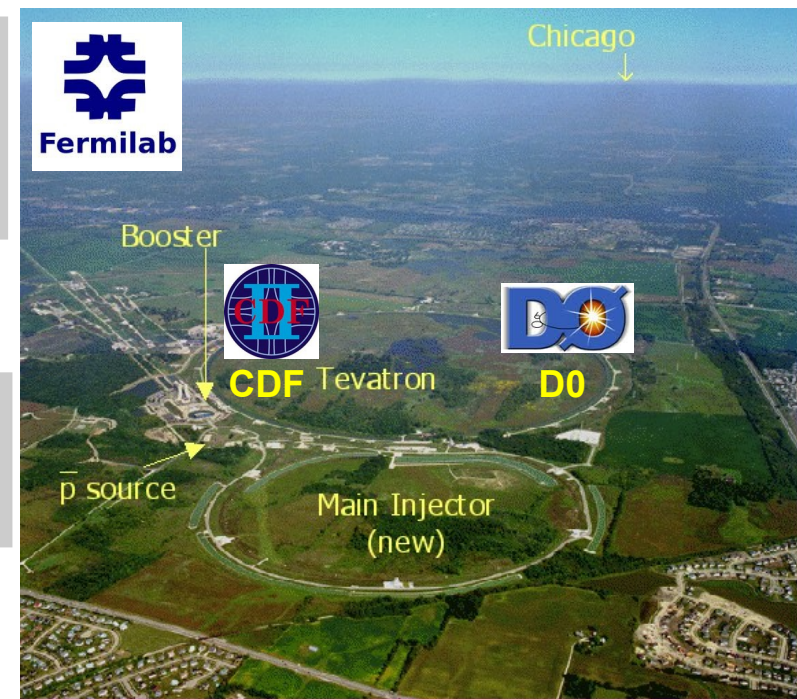


Hadron colliders

Tevatron: 1986 – 2011
 Collisions of **p anti-p**
 Run II: $E_{\text{cms}} = 1.96 \text{ TeV}$
 Run II: Record luminosity: $4.3 \times 10^{32} \text{ cm}^{-2}\text{s}^{-1}$

LHC: 2009 – present
 Collisions of **p-p, Pb-Pb, and p-Pb**
 $E_{\text{cms}} = 0.9, 2.36, 2.76, 5.02, 7, 8, 13 \text{ TeV}$
 Peak inst. Luminosity: $\sim 2 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$

HERA: 1992 – 2007
 Collisions of **e⁺-p, e⁻-p**
 HERA II: $E_{\text{cms}} = 319 \text{ GeV}$





Event rates at the LHC

Total cross section

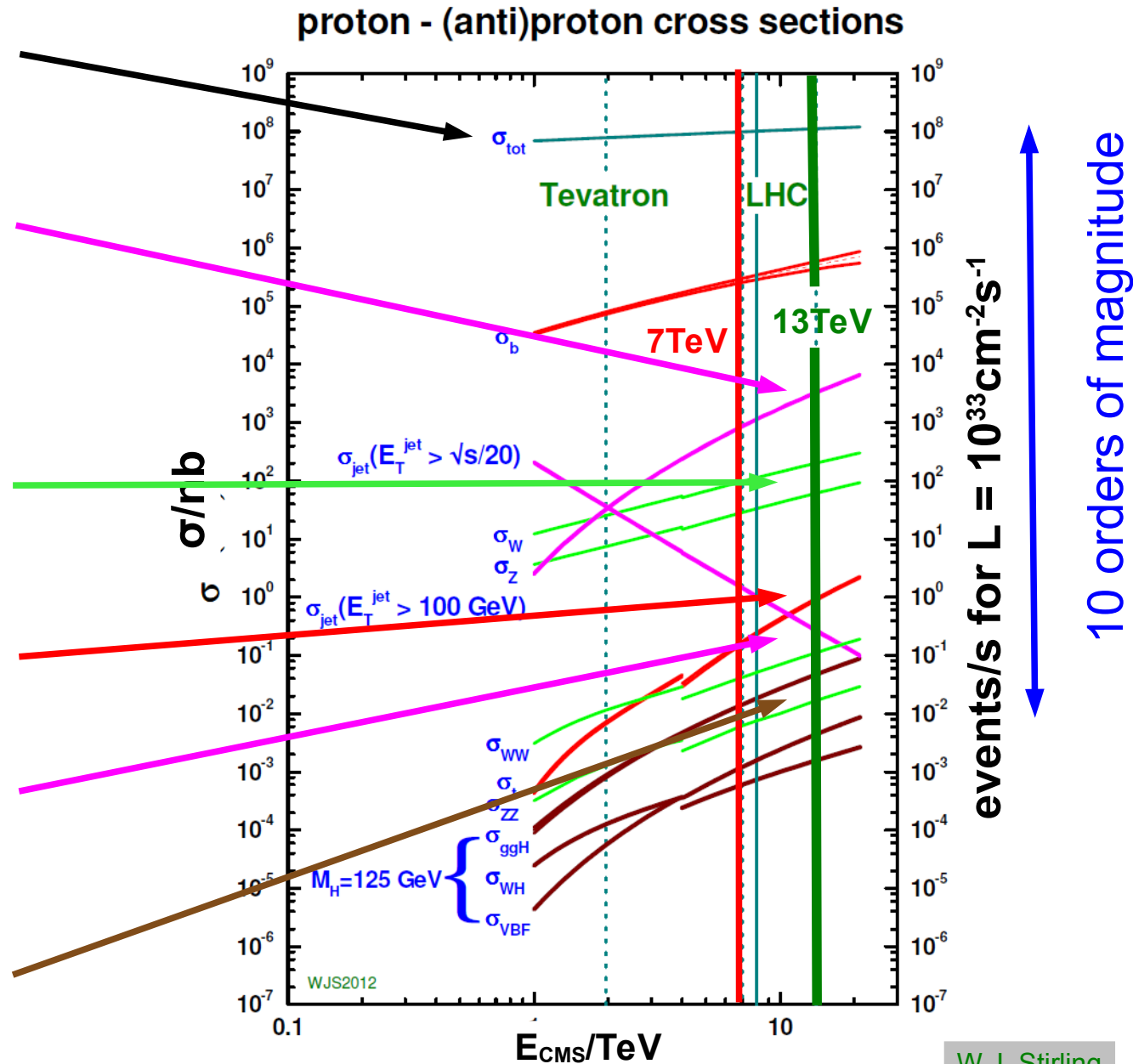
Jets: $\sigma_{\text{jet}}(E_T^{\text{jet}} > 100\text{GeV})$
 $\sim 2000 / \text{s}$

W & Z bosons: σ_W, σ_Z
 $\sim 200 / \text{s}, 50 / \text{s}$

Top quarks (σ_{tt})
 $\sim 1 / \text{s}$

Jets: $\sigma_{\text{jet}}(E_T^{\text{jet}} > 650\text{GeV})$
 $\sim 18 / \text{min}$

Higgs Bosonen ($\sigma_{\text{ggH}}, \sigma_{\text{WH}}, \sigma_{\text{VBF}}$)
 $\sim 150 / \text{h}$



W.J. Stirling

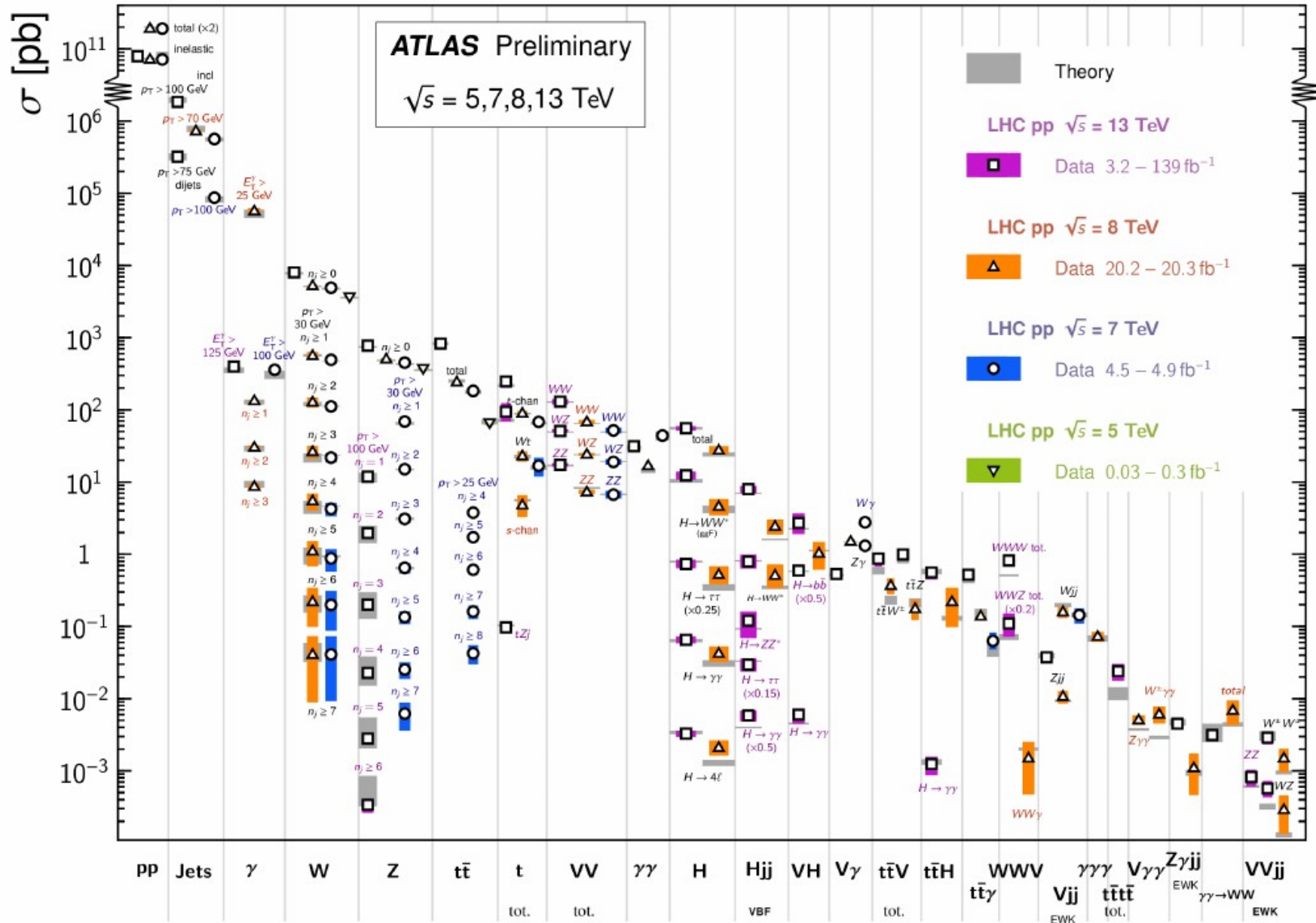


SM cross section data vs. theory



Standard Model Production Cross Section Measurements

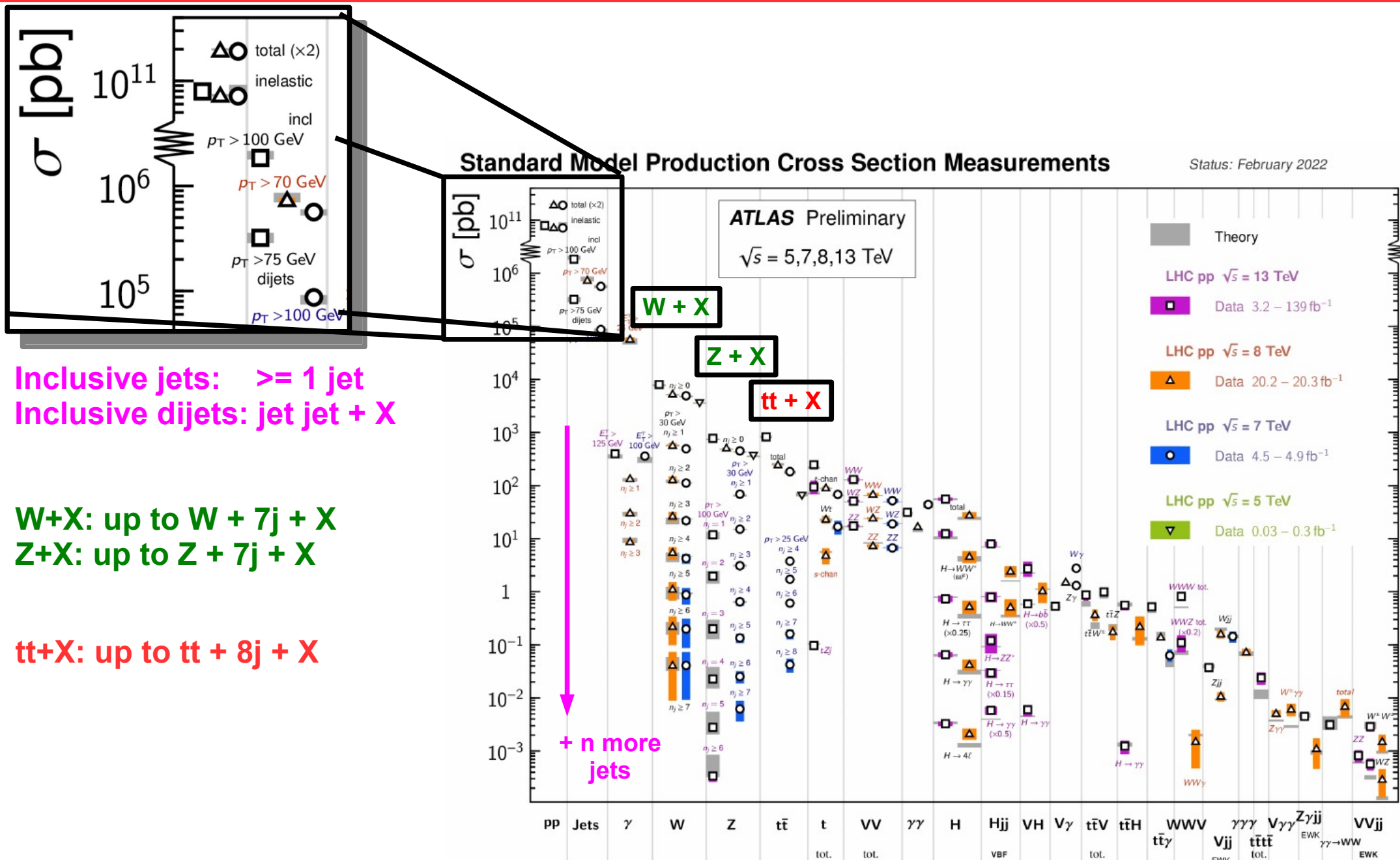
Status: February 2022



ATLAS



Many jets at the LHC



Inclusive jets: ≥ 1 jet
 Inclusive dijets: jet jet + X

W+X: up to W + 7j + X
 Z+X: up to Z + 7j + X

tt+X: up to tt + 8j + X



The ATLAS Detector



Inner Detector (ID) tracker:

- Si pixel and strip + transition rad. tracker
- $\sigma(d_0) = 15\mu\text{m}@20\text{GeV}$
- $\sigma/p_T \approx 0.05\%p_T \oplus 1\%$

Calorimeter

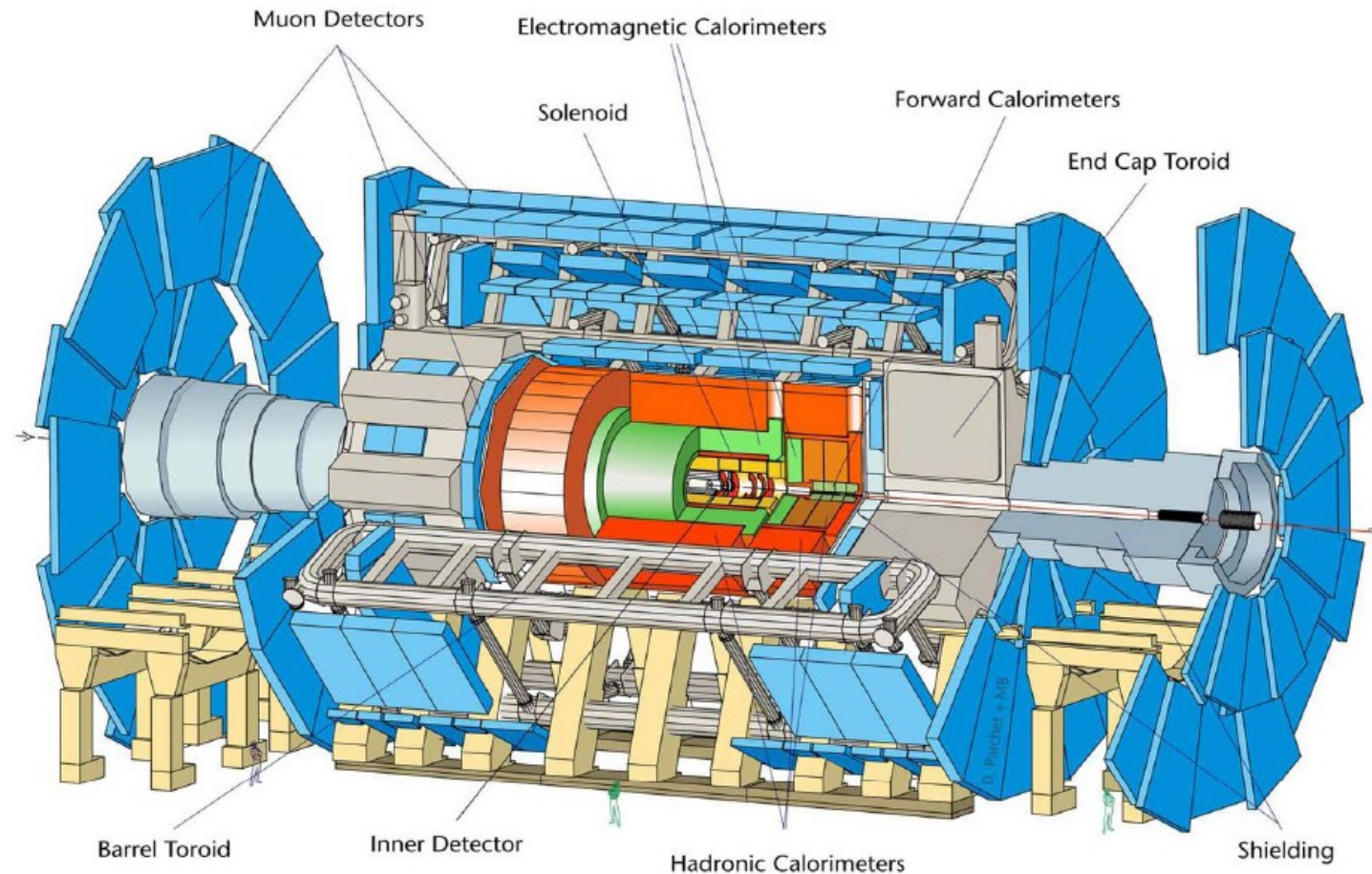
- Liquid Ar EM Cal, Tile Had. Cal
- EM: $\sigma_E/E = 10\%/\sqrt{E} \oplus 0.7\%$
- Had: $\sigma_E/E = 50\%/\sqrt{E} \oplus 3\%$

Muon spectrometer

- Drift tubes, cathode strips: precision tracking +
- RPC, TGC: triggering
- $\sigma/p_T \approx 2\text{-}7\%$

Magnets

- Solenoid (ID) $\rightarrow 2\text{T}$
- Air toroids (muon) \rightarrow up to 4T

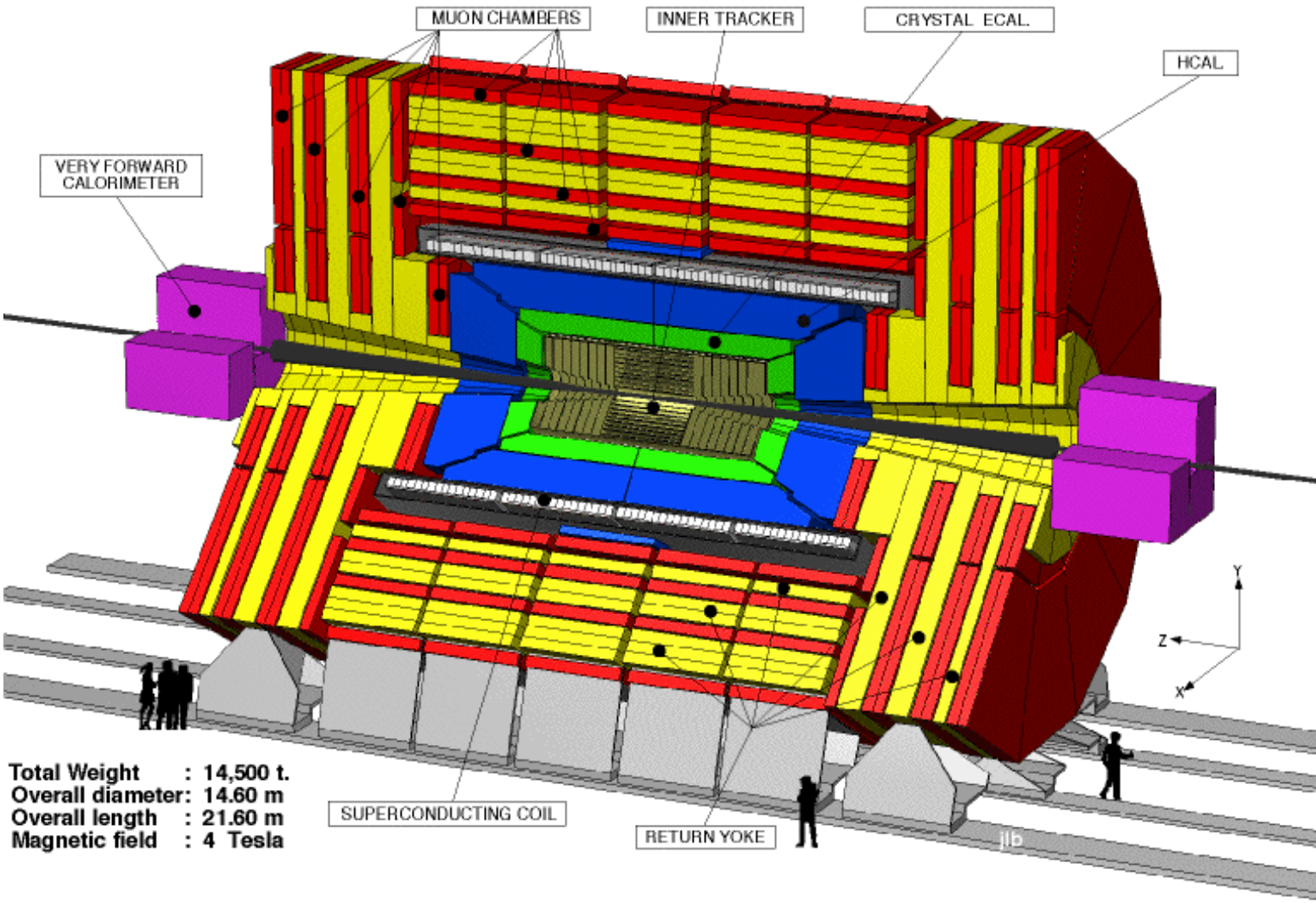


Full coverage for $|\eta| < 2.5$, calorimeter up to $|\eta| < 5$

See also JINST 3 2008 S08003



The CMS Detector



Total Weight : 14,500 t.
 Overall diameter: 14.60 m
 Overall length : 21.60 m
 Magnetic field : 4 Tesla

Inner detector (tracker):

- Si pixel & strip tracker
- $\sigma/p_T \approx 1-2\%$ (μ at 100 GeV)

Calorimeter:

- PbWO4 crystal ECAL, brass/scintillator HCAL
- ELM: $\sigma_E/E = 2.8\%/\sqrt{E} + 0.3\%$
- HAD: $\sigma_E/E = 100\%/\sqrt{E} + 5\%$

Muon system:

- Drift tubes, cathode strips, resistive plate chambers
- $\sigma/p \approx 10 - 50\%$ (muon alone)
- $\approx 0.7 - 20\%$ (with tracker)

Magnet:

- Solenoid $\rightarrow 3.8T$

See also:
 PTDR | LHCC-2006-001,
 JINST 3 2008 S08003



Jet analysis uncertainties



- **Experimental uncertainties**
(~ in order of importance):

- ➔ **Jet Energy Scale (JES)**
 - ➔ Noise treatment
 - ➔ Pile-Up treatment
- ➔ Luminosity (1 - 4%)
- ➔ **Jet Energy Resolution (JER)**
- ➔ Trigger efficiencies
- ➔ Resolution in rapidity
- ➔ Resolution in azimuth
- ➔ Non-Collision background
- ➔ ...

- **Theoretical uncertainties:**

- ➔ **MHOU (scale variation)**
- ➔ **PDF uncertainty**
- ➔ **Non-perturbative corrections**
- ➔ **Electroweak corrections**
- ➔ **PDF parameterization**
- ➔ **Knowledge of $\alpha_s(M_z)$**
- ➔ ...



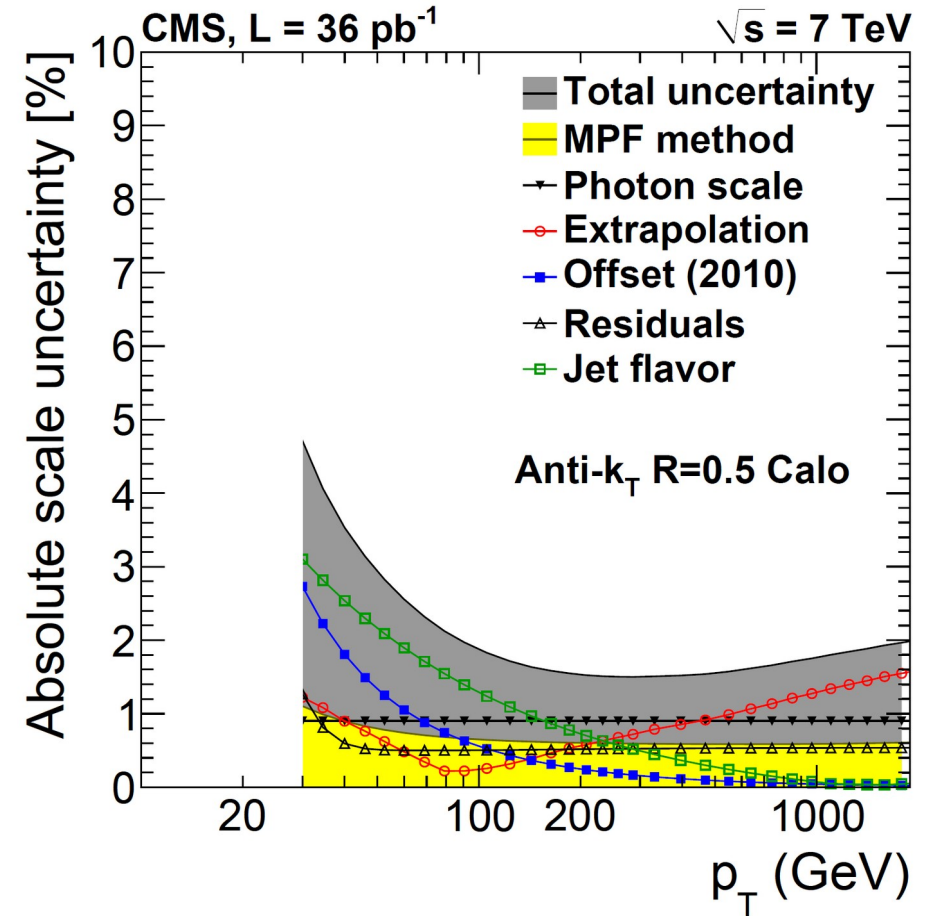
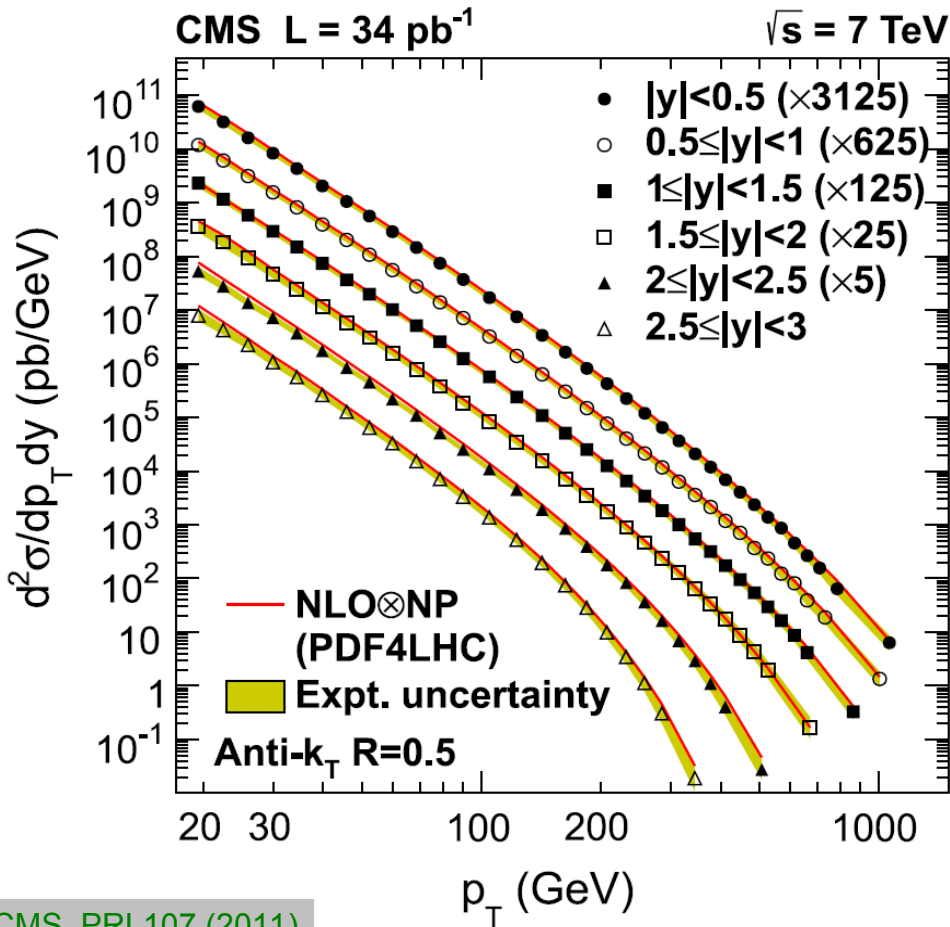
Why are JEC so important?



Steeply falling spectrum

$$\frac{d^2\sigma}{dp_T dy} \propto \frac{1}{p_T^{5-6}}$$

Factor 5 – 6 on uncertainty in energy scale, e.g. 2% → 10%



CMS, PRL107 (2011)



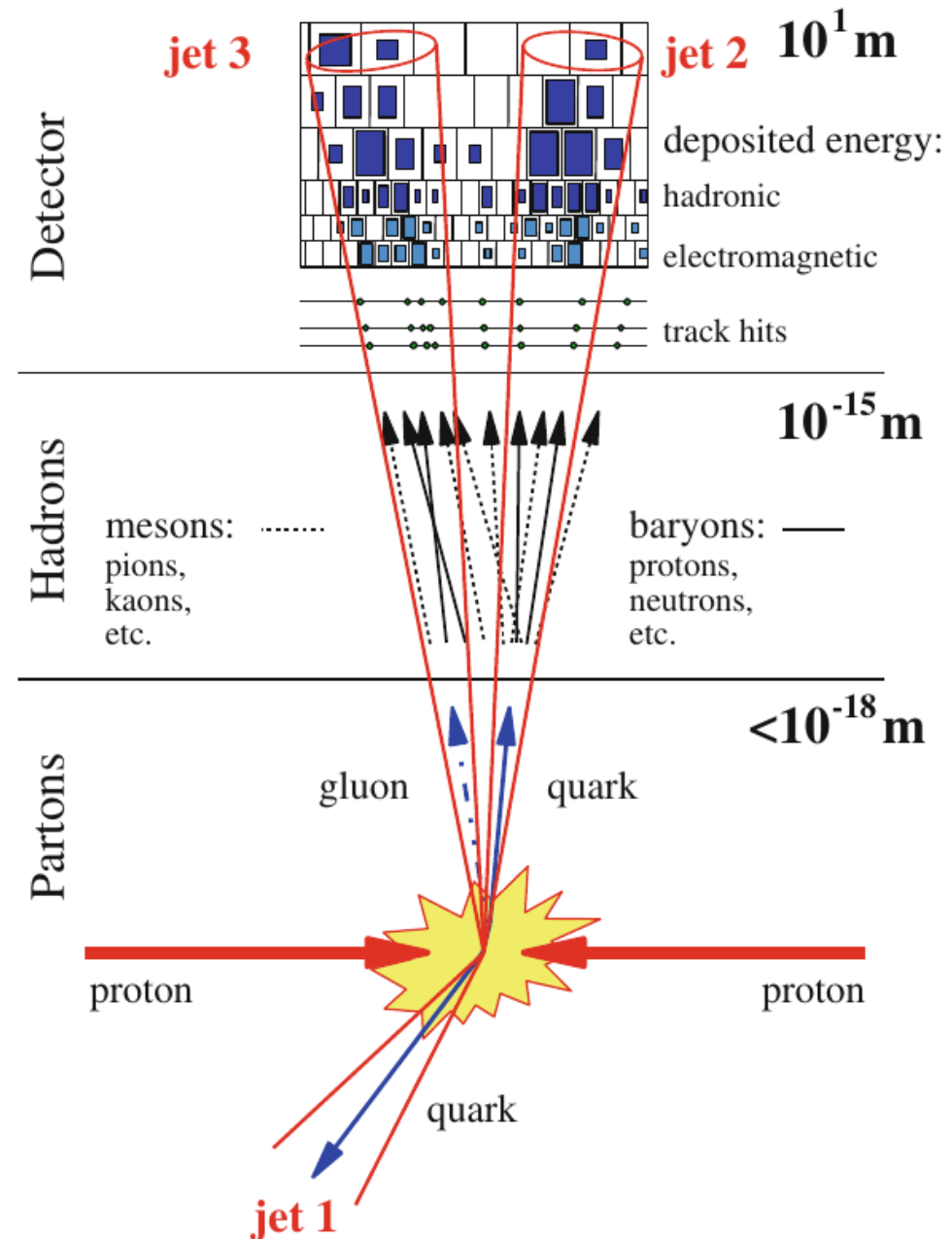
Jet reconstruction

Jet content:

- Tracks
- elm. calorimeter cells/clusters
- had. calorimeter cells/clusters

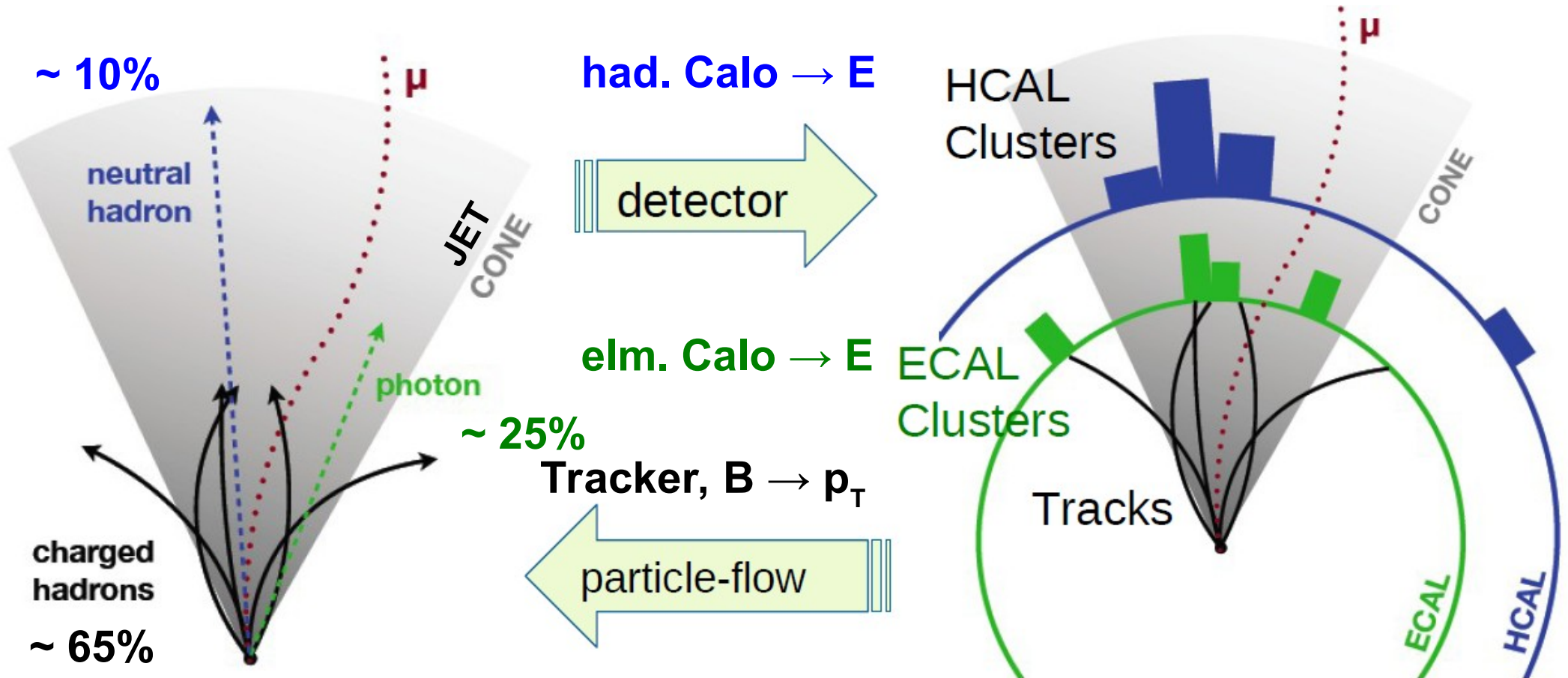
How to combine to get good estimate of original energy?

How to calibrate away remaining differences?





“Particle Flow” concept

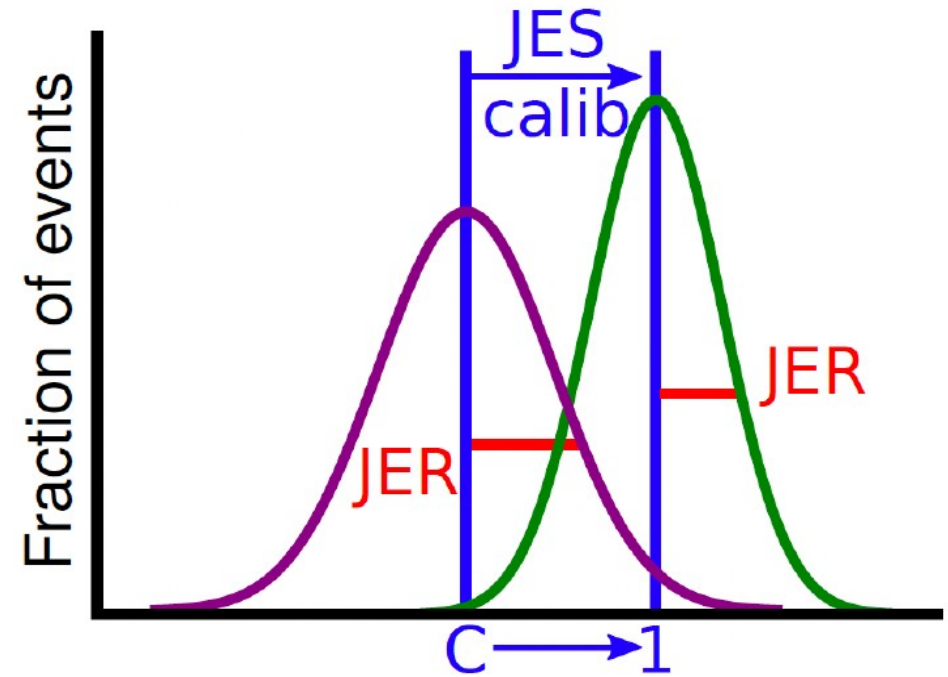
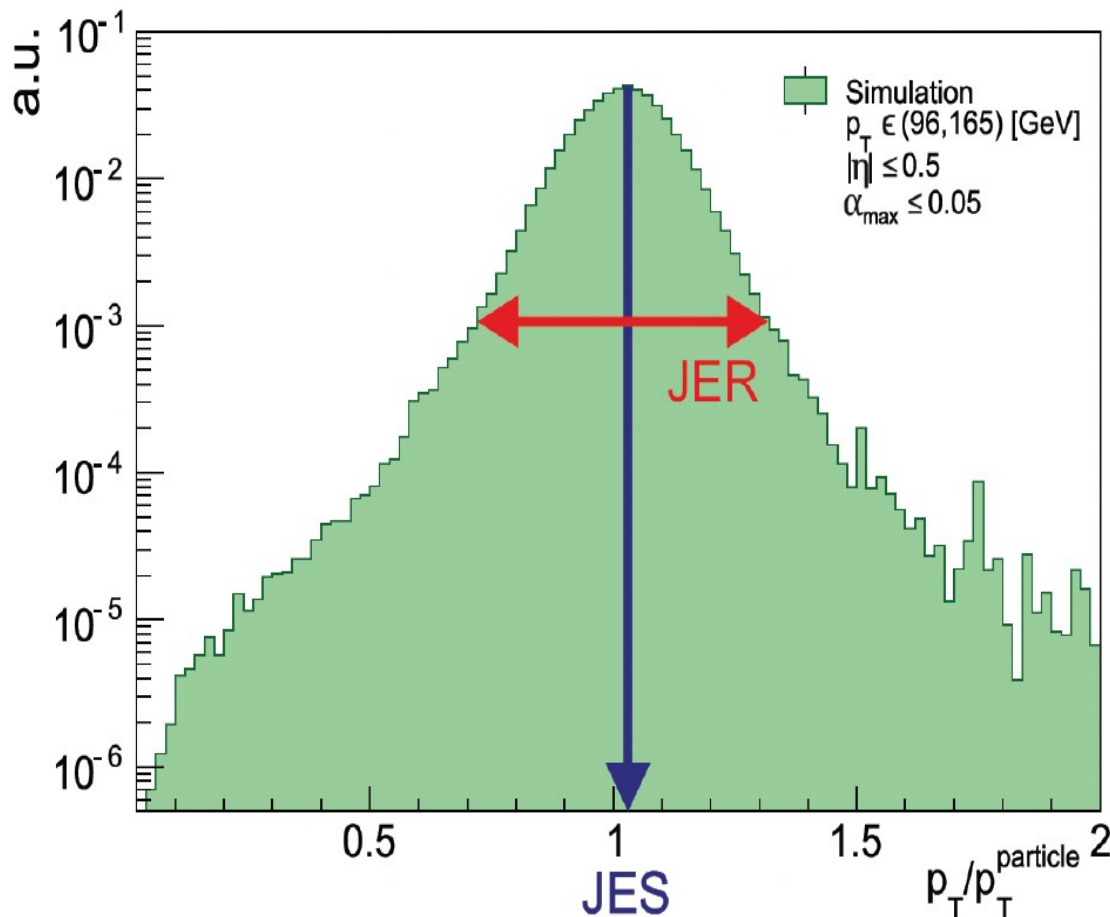


- + Combine measurements of various detector components
- + Consider specific detector properties wrt. particle type



Undo detector distortions

Use simulation to determine net shift and smearing of jet energies.
Use real data to correct for residual Biases.



Question:
What kind of real data can we take for this purpose?



Absolute calibration



- **Method 1: pT-balance against better known reference**
 - + Requires nicely balanced events
 - + **Insensitive to less well calibrated detector regions**
 - + **Sensitive to additional high-pT activity in events (2nd jets etc.)**

$$R_{\text{jet},pT} = \frac{p_{T,\text{jet}}}{p_{T,\text{ref}}}$$



Example picked up from ATLAS

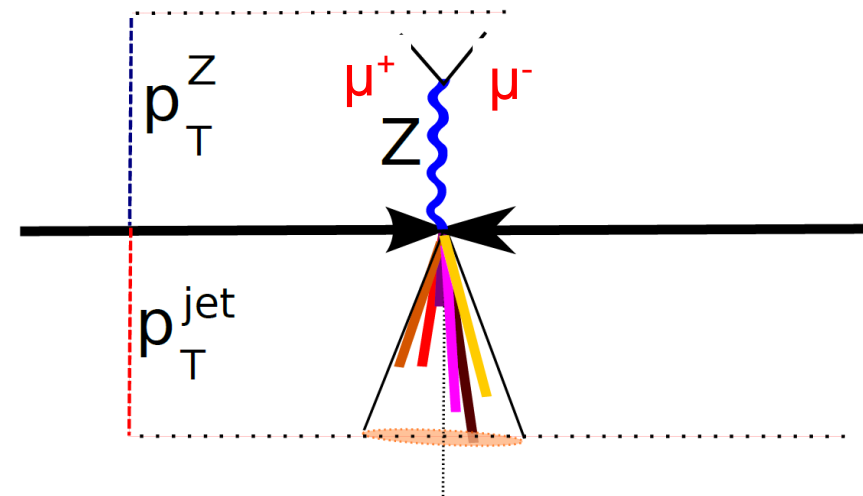
In-situ techniques used to validate JES and its uncertainty

- use well calibrated object(s) as reference for jet p_T
- compare balance of calibrated jets in data and Monte Carlo simulation

Techniques used in ATLAS:

- **Track-jets:** Compare calorimeter jets to track-jets
- **MPF method:** Employ MET projection to check γ and recoil balance
- **Photon-jet balance:** Balance p_T of γ and recoiling jet
- **Z-jet balance (2011):** Balance p_T of $Z \rightarrow ee$ with recoil jet [ATLAS-CONF-2011-159]
- **Multi-jet balance:** Balance high p_T jet with recoil system

Z-jet balance



Most precise method



- **Method 1: pT-balance against better known reference**

- Requires nicely balanced events
- **Insensitive to less well calibrated detector regions**
- **Sensitive to additional high-pT activity in events (2nd jets etc.)**

$$R_{\text{jet},pT} = \frac{p_{T,\text{jet}}}{p_{T,\text{ref}}}$$

- **Method 2: Missing transverse momentum projection fraction (MPF)**

- **No constraint to balanced events**
- **Requires reasonable calibration everywhere (except object to be calibrated)**

$$R_{\text{jet},\text{MPF}} = 1 + \frac{\vec{p}_{T,\text{miss}} \cdot \vec{p}_{T,\text{ref}}}{(p_{T,\text{ref}})^2}$$



Example picked up from ATLAS

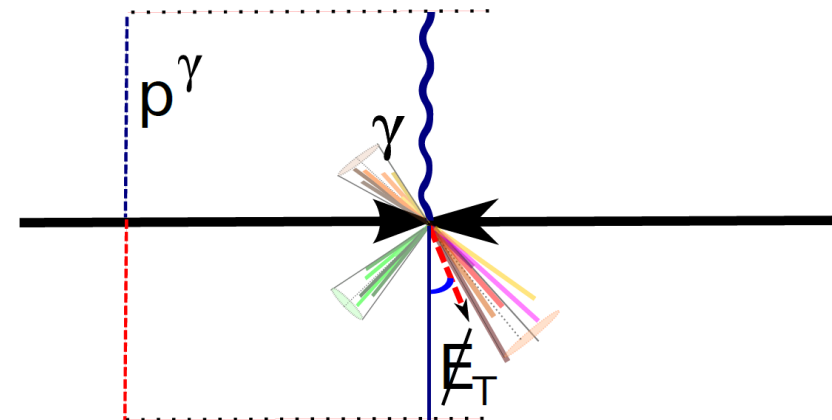
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MPF method





Absolute calibration



- **General data-based principle usually the same:**

Comparison of less precisely known with better known object!

- ➔ **Detector measurements are most precise for:**
 - ➔ **Combination of inner tracker and outer muon tracks → muons ($p_T > 3-5\text{GeV}$)**
 - ➔ **Inner tracker alone → isolated charged hadrons (p_T not too small → loops, or too large wrt. B field → straight tracks)**
 - ➔ **Inner tracker and electromagnetic calorimeter → electrons**
 - ➔ **Electromagnetic calorimeter alone → photons**
- ➔ **Propagation of more precise measurements:**
 - ➔ **From medium to lower or higher p_T :
Best precision with $Z(\rightarrow \mu\mu) + 1$ jet → abs. calibration in central detector**
 - ➔ **From central detector outwards, from high event rates to areas with small event numbers:
Best “statistics” (Number of events) with jets → dijet extrapolation from central detector → towards higher rapidities**
 - ➔ **Only possibility at highest p_T →
Balance of two or more low p_T jets against one high- p_T jet**



Take home message



General data-based principle usually the same:

Comparison of less precisely known with better known object!

- Detector measurements are most precise for:
 - Combination of inner tracker and electromagnetic calorimeter → **neutrons** ($p_T > 3-5\text{GeV}$)
 - Inner tracker → **neutrons** (p_T not too large wrt. B field → straight tracks)
 - Inner tracker and electromagnetic calorimeter → **electrons**
 - Electromagnetic calorimeter alone → **photons**
- Propagation of more precise measurements:
 - From medium to lower or higher p_T :
Best precision (central detector) + 1 jet → abs. calibration in central detector
 - From central detector to high rapidities: high event rates to areas with small event numbers:
Best “statistics” (Number of events) with high event rates → dijet extrapolation from central detector → towards high rapidities
 - Only possibility at highest p_T →
Balance of two or more low p_T jets against one high- p_T jet

Calibrate less precisely measured object against more precisely measured one!

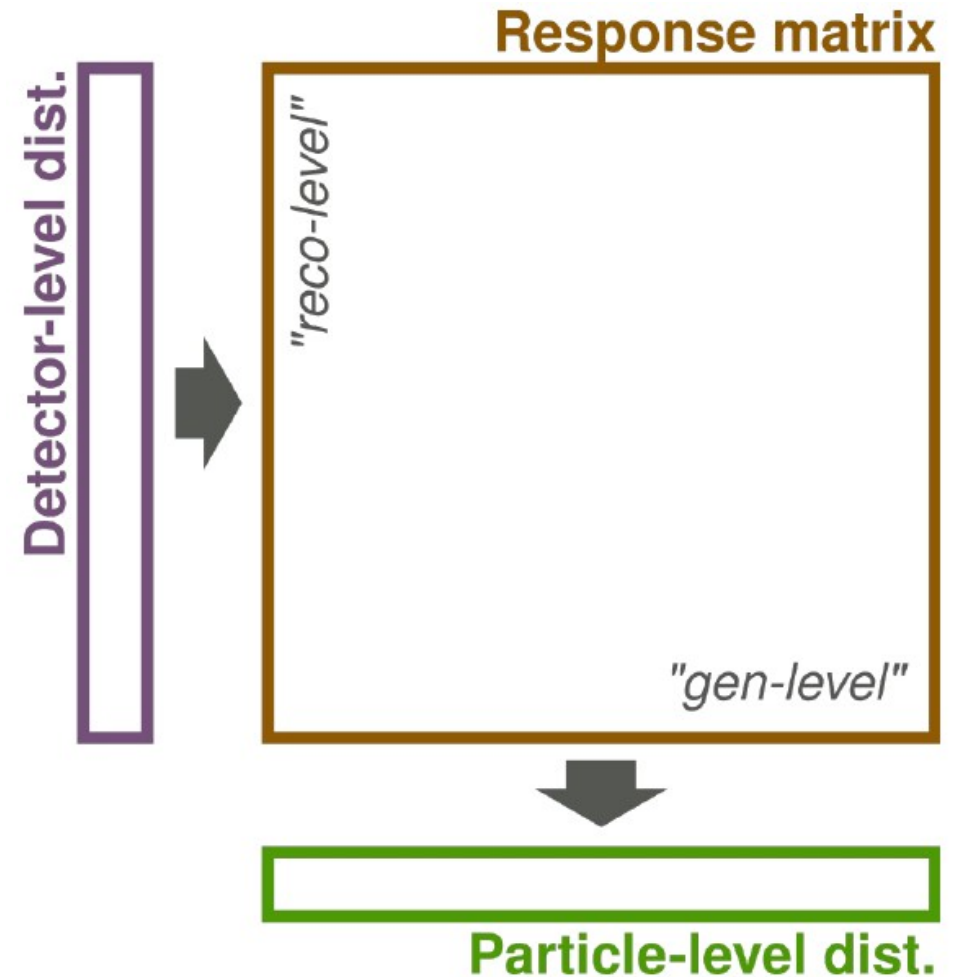
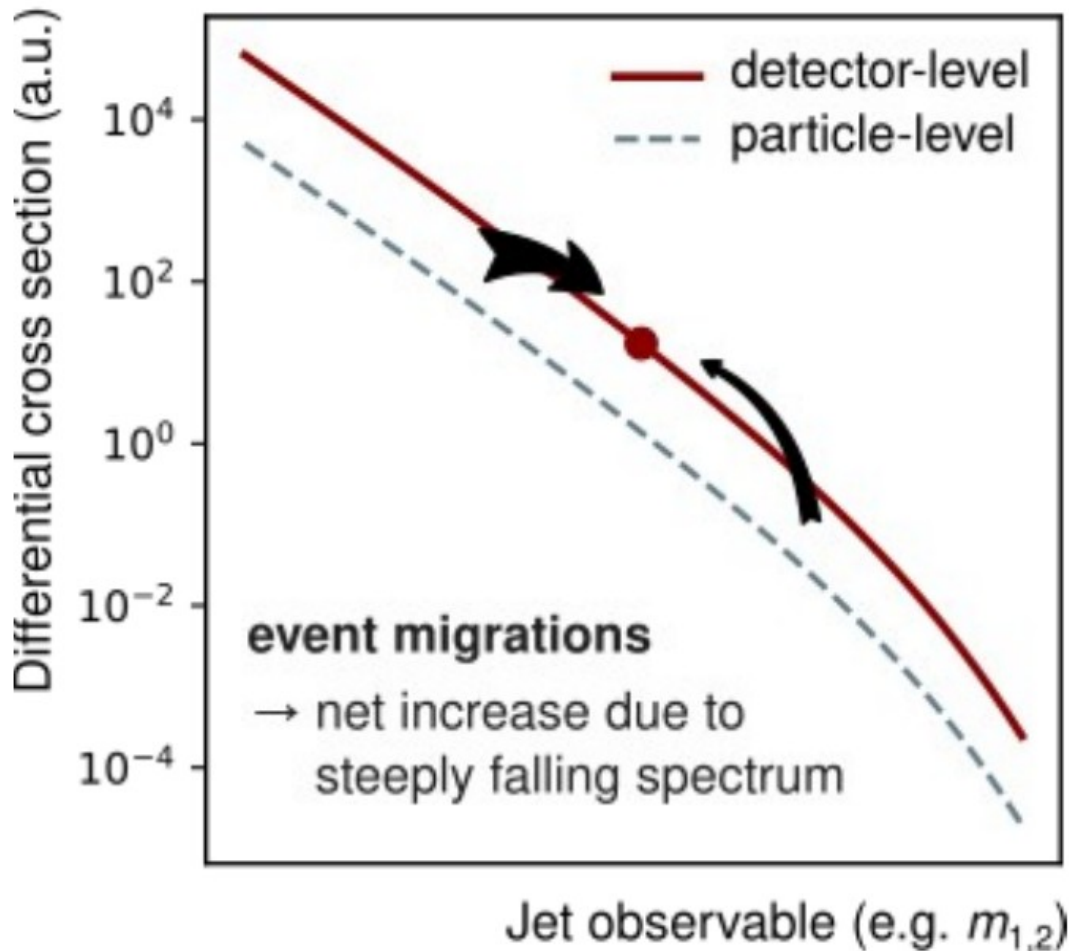
Calibrate less well-calibrated detector region against better calibrated one!



Jet energy resolution

Net effect of JER often looks like increase in cross section

Use response matrix from MC simulation to unfold the smearing.





Response matrix

